Responsive Affirmative Action in School Choice

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November 7, 2014

Matching Theory

Provide solutions to several resource allocation problems.

- assigning students to schools.
- assigning doctors to hospitals' residency positions.
- assigning human organs for transplant to recipients.

Design program for school choice

"Better" rules to assign students to schools.

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But, how about affirmative action?

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A recent theoretical result (Kojima, 2012): Current affirmative action policies may hurt some minority students without benefiting another minority student.

Main Contributions

1. Situations at which no minority student benefits from affirmative action are pervasive.

2. Propose a new rule which achieves affirmative action and never hurts a minority student without benefiting another minority student.

School choice problem with affirmative action

Minority students:	S^m	
Majority students:	S^M	
Schools:	C	
School capacities:	$(q_c)_{c \in C}$	
Priorities:	$(\succeq_c)_{c \in C}$ over S	complete, transitive, antisymmetric
Preferences:	$(R_s)_{s \in S}$ over $C \cup \{s\}$	complete, transitive, antisymmetric

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A problem: $(S^m, S^M, C, q, \succeq, R, r)$

The set of all problems: $\mathcal{P}$

A matching μ: Assignment of students to schools which respects the capacities.

$\mu : S \cup C \to 2^{S \cup C}$ such that

i. $\forall s \in S$, $\mu(s) \in C \cup \{s\}$,

ii. $\forall c \in C$, $\mu(c) \subseteq S$ and $|\mu(c)| \leq q_c$, and

iii. $\forall s \in S, c \in C$, $\mu(s) = c \Leftrightarrow s \in \mu(c)$.

Set of all matchings: $\mathcal{M}$

A rule φ: $\mathcal{P} \to \mathcal{M}$

Fairness without affirmative action

A matching μ is fair at $(S^m, S^M, C, q, \succeq, R)$ if

1. No priority violation. $\nexists\ (s, c)$ such that
   (i) $c\ P_s\ \mu(s)$ and
   (ii) $\exists\ s' \in \mu(c)$ such that $s \succ_c s'$.

2. Outside option lower bound. $\forall\ s,\ \mu(s)\ R_s\ s$.

3. Non-wastefulness. $\exists (s, c) : c\ P_s\ \mu(s) \implies |\mu(c)| = q_c$.

Fairness w.r.t. the quota

r_c : number of seats available only to minority

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Three modifications:

1. Majority-quota requirement: $\forall\ c \in C,\ |\mu^M(c)| \leq q_c - r_c$.

2. M-violation at full quota: $\exists\ m \in S^m, M \in S^M, c \in C$ such that

   (i) the priority of M is violated by m at c and
   (ii) $|\mu^M(c)| = q_c - r_c$.

   Allow M-violation at full quota.

3. Non-wastefulness: If $\exists (s, c) : c\ P_s\ \mu(s)$, then

   either $s \in S^m$ and $|\mu(c)| = q_c$
   or $s \in S^M$ and $|\mu^M(c)| = q_c - r_c$.

Fairness w.r.t. the reserve

r_c : minority reserve, but can be assigned to a majority if no minority is interested.

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Two modifications:

1. *m*-violation at unexhausted reserve: $\exists\ m \in S^m, M \in S^M, c \in C$ such that

   (i) $c\ P_m\ \mu(m)$, $M \in \mu(c)$, $M \succ_c m$, and
   (ii) $|\mu^m(c)| < r_c$.

   Not allow *m*-violation at unexhausted reserve.

2. *M*-violation at unexceeded-reserve: $\exists\ m \in S^m, M \in S^M, c \in C$ such that

   (i) the priority of M is violated by m at c and
   (ii) $|\mu^m(c)| \leq r_c$.

   Allow *M*-violation at unexceeded-reserve.

Responsiveness to affirmative action

A rule φ is minimally responsive if

$$\forall\ (S^m, S^M, C, q, \succeq, R, r) \text{ and } r' \geq r\text{:}\quad \begin{array}{c}\varphi(R, r) \text{ does not Pareto dominate } \varphi(R, r') \\ \text{for the minority}\end{array}$$

a stronger affirmative action never hurts a minority student
without benefiting another minority student.

Strategy-proofness

A rule φ is strategy-proof if

$\nexists (S^m, S^M, C, q, \succeq, R, r)$, $s \in S$, and R'_s such that

$$\varphi_s(S^m, S^M, C, q, \succeq, R_{-s}, R'_s, r) \; P_s \; \varphi_s(S^m, S^M, C, q, \succeq, R, r)$$

No student can ever gain
by misreporting her preferences.

Impossibilities

No rule is fair w.r.t. the quota and minimally responsive (Kojima, 2012)

Impossibilities

Theorem

$$\nexists \text{ rule} \quad \begin{array}{c} \textit{fair w.r.t. the reserve} \\ \& \\ \textit{minimally responsive} \end{array} .$$

Degree of the impossibilities

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Fix $(S^m, S^M, C, \succ, q)$.

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For which $(S^m, S^M, C, \succ, q)$ profiles the impossibilities disappear?

Theorem

Assume that $|S^m| > max(q_c)_{c \in C}$.

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$$\exists \text{ rule} \quad \begin{array}{c} \textit{fair w.r.t. the quota} \\ \& \\ \textit{minimally responsive} \end{array} \iff \begin{array}{cc} \underline{\succ_c} & \underline{\succ_{c'}} \\ m & m'' \\ m' & m \\ m'' & m' \\ \vdots & \vdots \\ M & M' \\ M' & M'' \\ M'' & M \\ \vdots & \vdots \end{array} \quad \cdots$$

Theorem

Assume that $\forall c, c'$, $|S^m| > q_c + q_{c'}$.

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$$\exists \text{ rule} \quad \begin{array}{c} \text{fair w.r.t. the reserve} \\ \& \\ \text{minimally responsive} \end{array} \quad \Longleftrightarrow \quad \text{for each } M \quad \begin{array}{c} \exists \text{ at least } |S^m| - 1 \text{ minority} \\ \text{each having} \\ \text{priority over } M \\ \text{at at least } |C| - 1 \text{ schools} \end{array}$$

The impossibilities disappear

only when

we do not need affirmative action!

DA with minority reserves, DA^m (Hafalir et al. 2013)

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However, DA^m is not minimally responsive.

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Theorem

Let $R \in \mathcal{R}^S$ and $r \leq r'$.

No minority student prefers $DA^m(R, r')$ to $DA^m(R, r)$ $\Longrightarrow$ *No majority student prefers $DA^m(R, r')$ to $DA^m(R, r)$*

Why is DA^m not minimally responsive?

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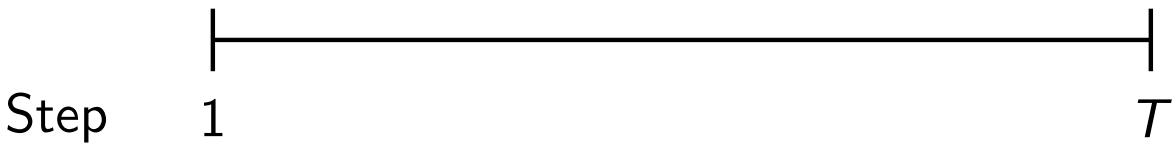

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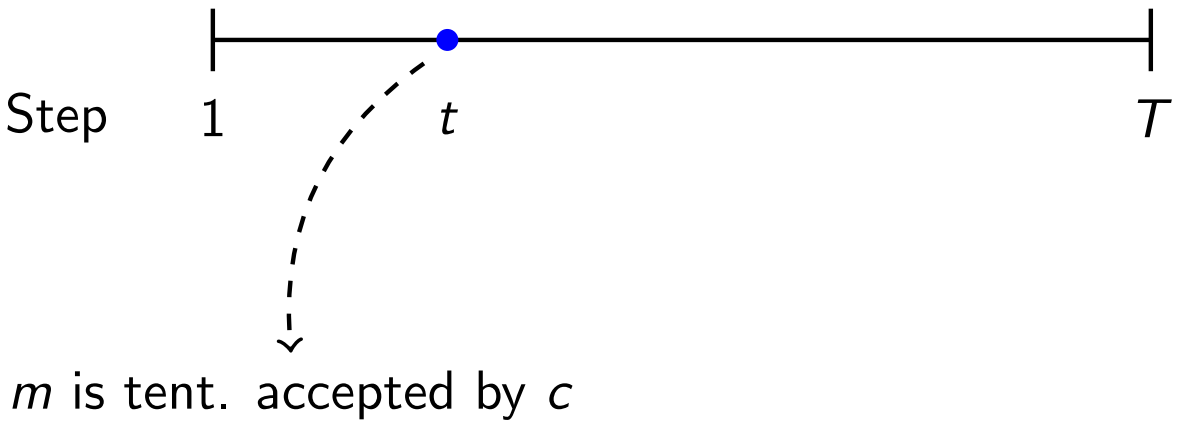

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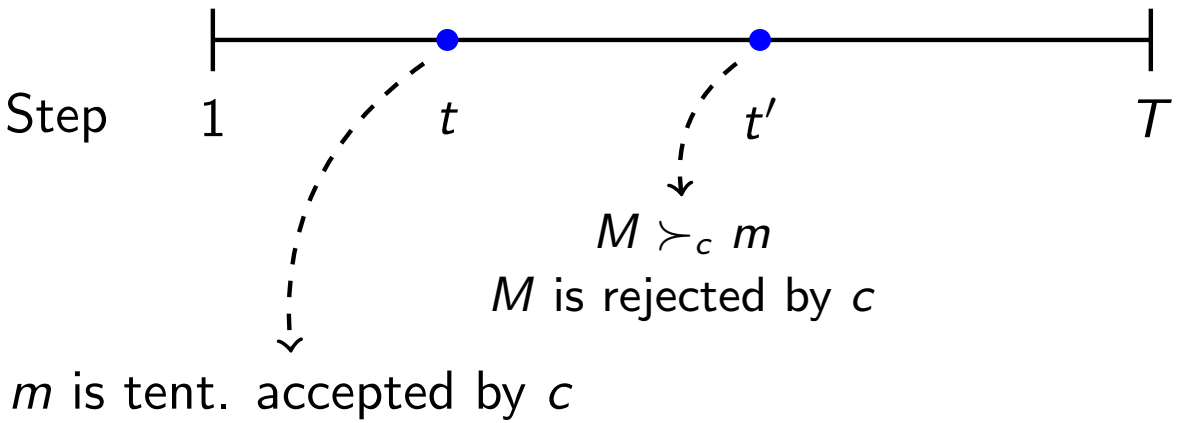

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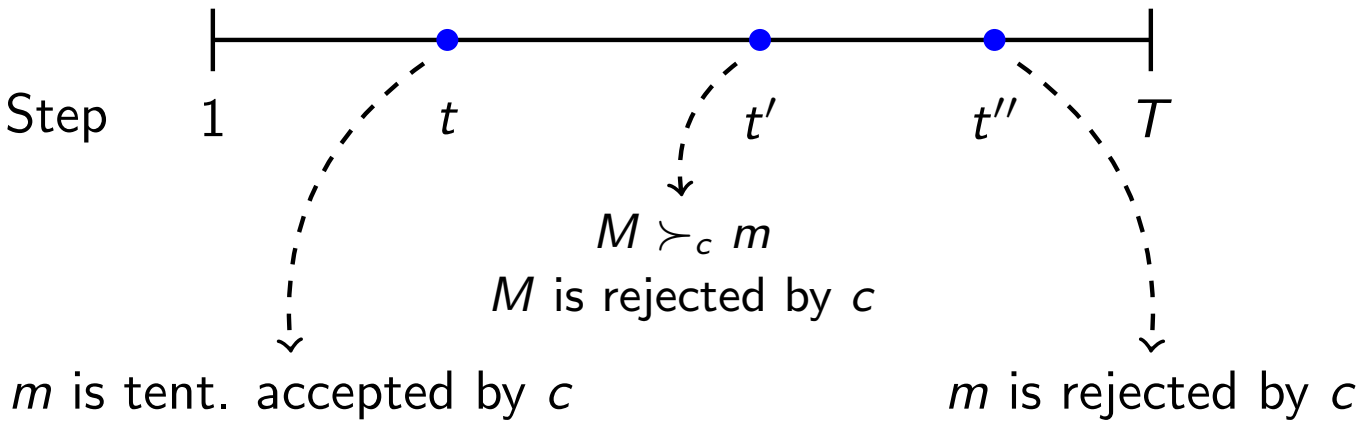

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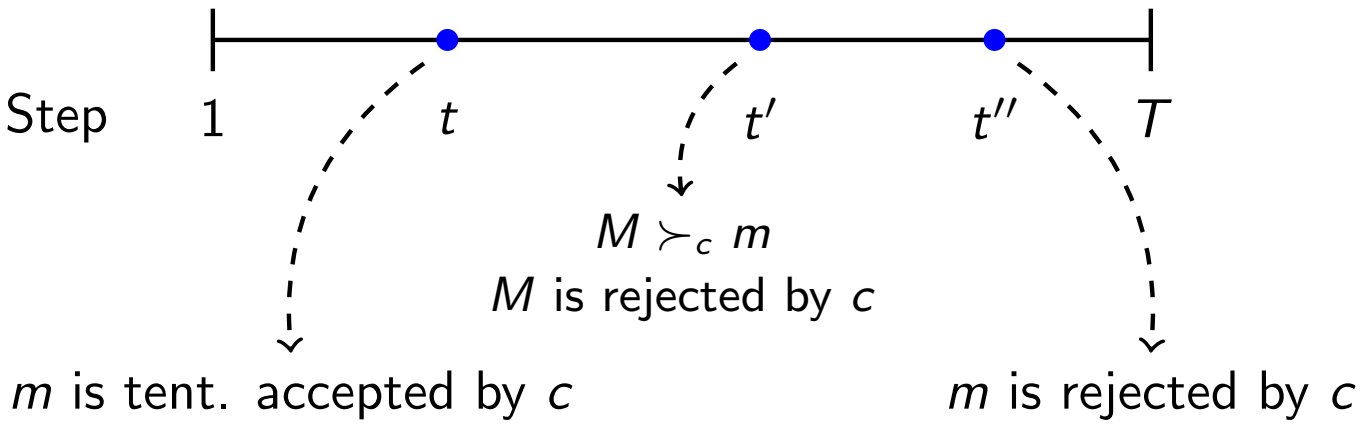

m is an interferer for c at Step t''

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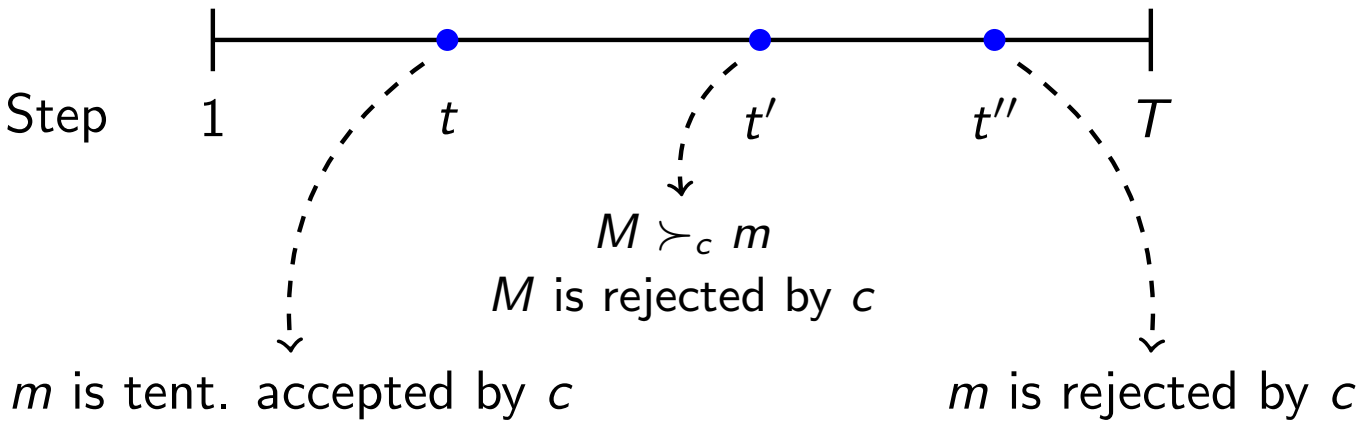

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Analogous to "interrupter" in Kesten (2010).

Modified DA with minority reserves, MDA^m

- Run DA^m. Determine last step at which there is interferer.

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Stops in finite time.

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Analogous to "EADAM" in Kesten (2010).

Theorem

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MDA^m is not fair w.r.t. the reserve.

Fairness w.r.t. the reserve

Two modifications:

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Fairness w.r.t. the reserve

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Fairness w.r.t. the weak-reserve

Two modifications:

1. Not allow the priority of m to be violated at c at unexhausted reserve only if there is $\mu \in F^{reserve}$ such that $m \in \mu(c)$.
2. Allow M-violation at unexceeded-reserve.

Theorem

MDA^m is fair w.r.t. the weak-reserve.

Theorem

At each problem, either MDA^m Pareto dominates DA^m or gives the same assignment.

Strategic properties

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$\nexists$ *rule* *fair w.r.t. the weak-reserve, minimally responsive, & strategy-proof* .

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MDA^m is not strategy-proof.

A majority or a minority can be better off by manipulating.

μ is **fair on the no–affirmative-action side** if:

1. No students' priority is violated at any school by any student of the same type,
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μ is **minimally fair** if:

1. μ is fair on the no–affirmative-action side,
2. there are no $m \in S^m$, $M \in S^M$, and $c, c' \in C$ such that
   1. $\mu(m) = c$ and $\mu(M) = c'$,
   2. $c' \; P_m \; c$, $|\mu^m(c')| < r_{c'}$, $r_c = 0$, and $M \succ_c m$,
   3. $\mu_{m \leftrightarrow M}$ is fair on the no–affirmative-action side.

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Theorem

No rule is minimally fair, minimally responsive, and strategy-proof.

Conclusion

A new rule, MDA^m: minimally responsive, fair w.r.t. the weak-reserve, and Pareto dominates DA^m.

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Limitation: Not strategy-proof.

Thank you!