Fair Implementation of Diversity in School Choice

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WZB Berlin Social Science Center

Matching in Practice Workshop
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Table of Contents

1 Introduction

2 Model

3 Comparative Analysis

4 Incentives

5 Conclusion

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2 Model

3 Comparative Analysis

4 Incentives

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Motivation

- School Districts: Objectives regarding how students of different types should be distributed across schools.
  - Desegregation efforts in the US.
  - Promote ethnic diversity in school cohorts.
  - Promote socioeconomic integration.

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- School Districts: Objectives regarding how students of different types should be distributed across schools.
  - Desegregation efforts in the US.
  - Promote ethnic diversity in school cohorts.
  - Promote socioeconomic integration.
- Solutions found in the recent School Choice literature: give higher priority to certain types of students.
  - Example: give higher priority to minority students for a subset of each school's seats.

Motivation

- Problem: they may yield segregated assignments for some familiar students' preference profiles.

Contributions

- A new School Choice mechanism — School-Proposing Diversity (SP-Div) — based on the School Proposing Deferred Acceptance algorithm, which:

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  - Maximizes the satisfaction of the diversity objectives among the set of fair assignments.

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- A new School Choice mechanism — School-Proposing Diversity (SP-Div) — based on the School Proposing Deferred Acceptance algorithm, which:
  - Produces assignments that satisfy a fairness criterion which includes the diversity objectives as an element of fairness.
  - Maximizes the satisfaction of the diversity objectives among the set of fair assignments.
  - Doesn't have to rely on the distribution of types on the population of students satisfying some necessary condition for those results to hold.

Contributions

- Recognize that a welfare property of the School Proposing Deferred Acceptance algorithm (that it maximizes the "schools' welfare" among the stable assignments) can be used as a tool to select the assignment that maximizes the satisfaction of the diversity objectives if we design a choice function to be used by the schools that incorporates a "preference" for satisfying those objectives.

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- Analytical results comparing assignments generated by the SP-Div and the student-proposing version of that mechanism in how much segregation there is in the assignments generated by them.

Policymakers' Objectives

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  - School desegregation policies in the U.S.

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- Concept introduced by Gale and Shapley (1962).
- Two procedures:
  - Student-Proposing Deferred Acceptance: student-optimal stable matching
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  - There is no stable mechanism that is immune to manipulation by schools (Roth, 1985)

Stable Matching and School Choice

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  - Abdulkadiroğlu and Sönmez (2003), Abdulkadiroğlu et al (2006) and Abdulkadiroğlu et al (2009).
- When concerns about diversity and affirmative action were introduced, that choice persisted
  - Abdulkadiroğlu and Sönmez (2003), Echenique and Yenmez (2012), Kominers and Sönmez (2012), Hafalir et al. (2013),Westkamp (2013).

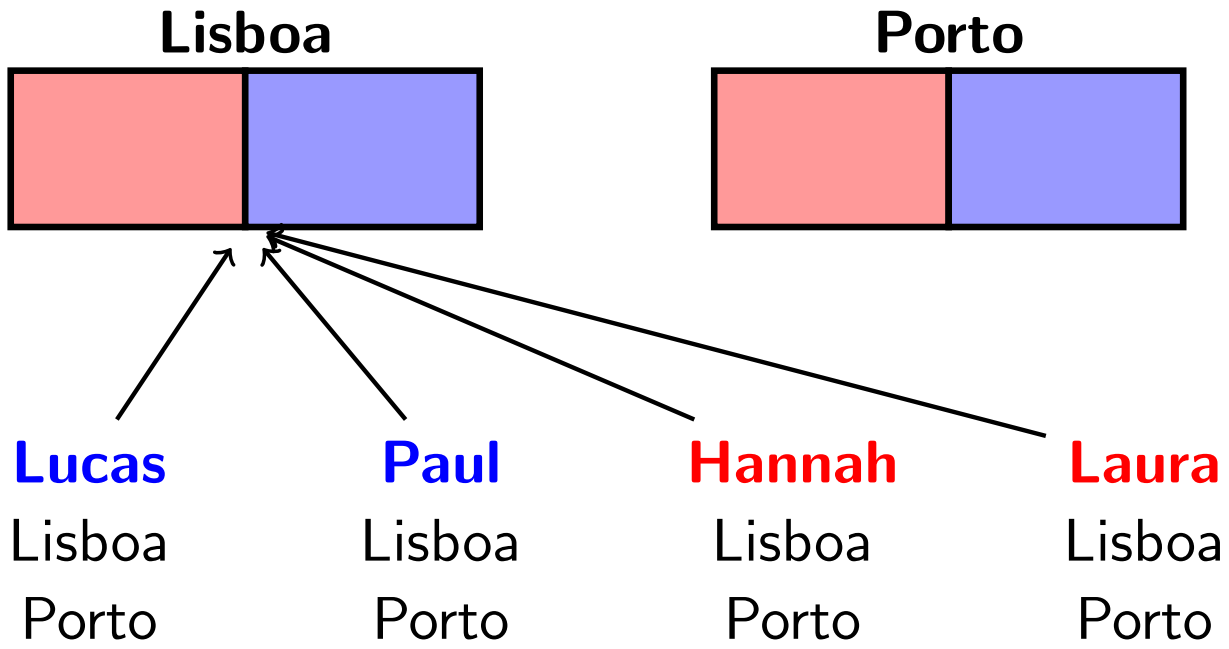
Student-Proposing vs School-Proposing

Student-Proposing Deferred Acceptance

Lucas	**Paul**	**Hannah**	**Laura**
Lisboa	Lisboa	Lisboa	Lisboa
Porto	Porto	Porto	Porto

Student-Proposing vs School-Proposing

Student-Proposing Deferred Acceptance



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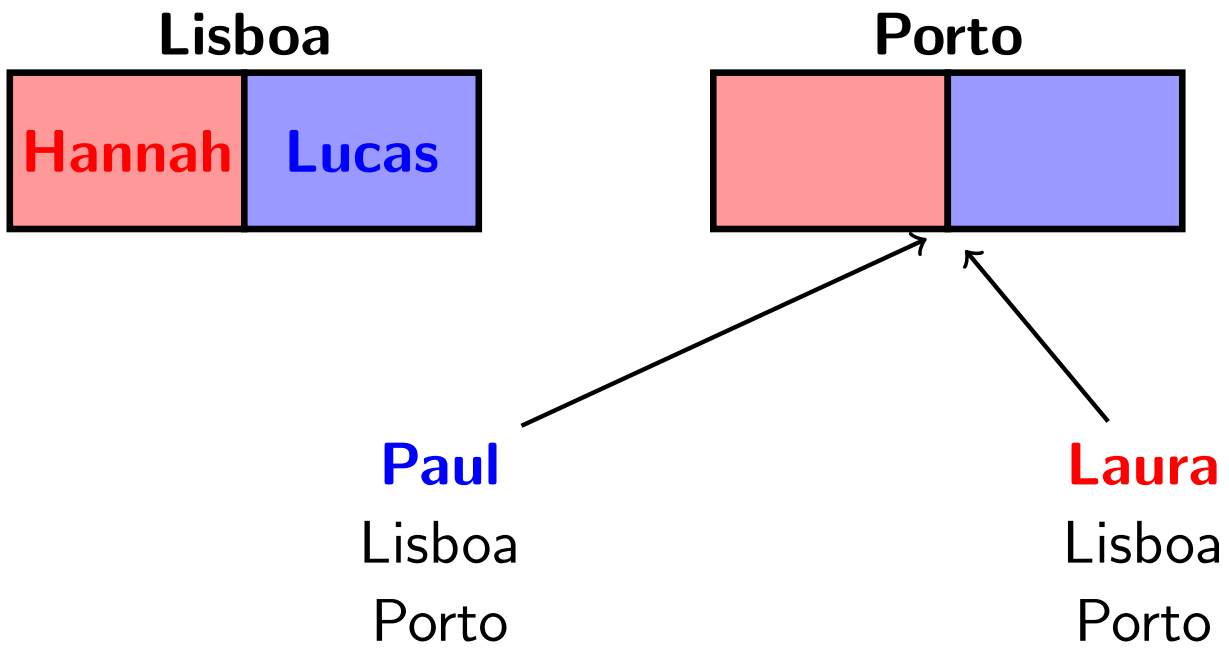

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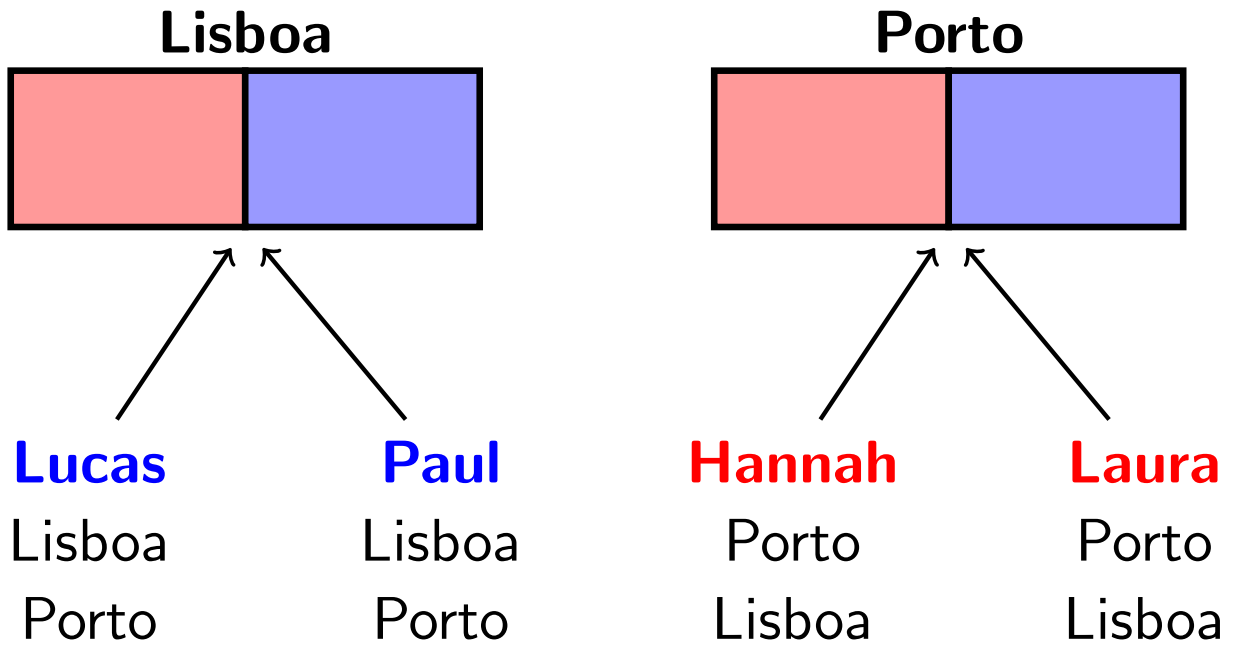

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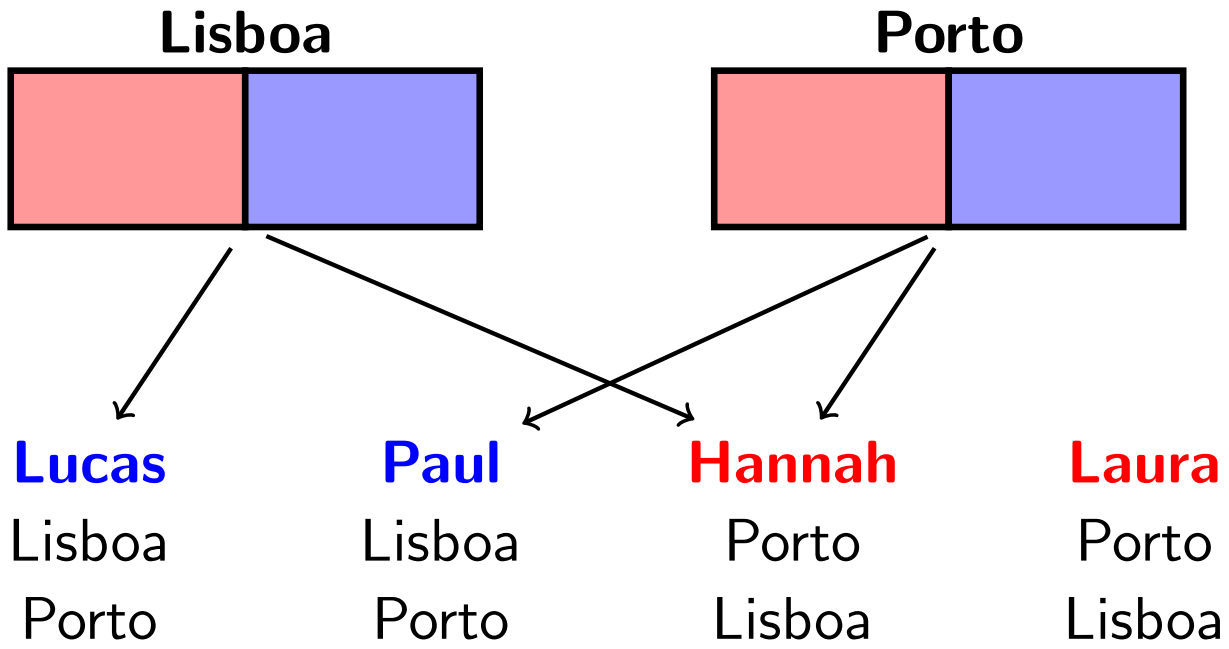

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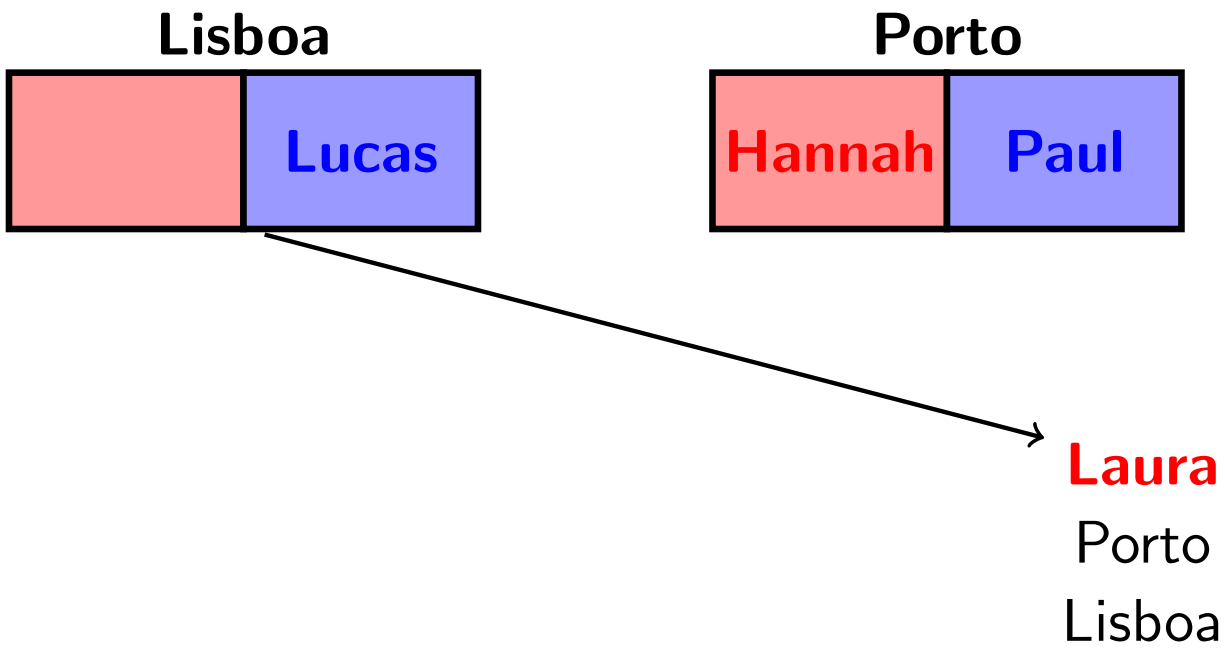

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8. Schools' **priority profile** $\succ_C = \left(\succ_{c_1}, \ldots, \succ_{c_{|C|}}\right)$.

Fairness

Balinski and Sönmez (1999)

A student s **justifiably envies** a student s' in school c if s prefers school c to her own and $s \succ_c s'$. An assignment is **fair** if no student justifiably envies another.

Fair with Diversity

Definition

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2. There is a student s', assigned to c, where $s \succ_c s'$ and switching s' to s would not make any diversity objective at c satisfied to a lesser extent.

An assignment is **fair with diversity** if it is *individually rational*, *non-wasteful* and no student *justifiably demands* a seat in any school.

Fair with Diversity

Proposition

There are problems in which all assignments that satisfy the diversity objectives are **not** *fair with diversity.*

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Let $>^{\underline{q}}$ be the order over the set of assignments such that $\mu' >^{\underline{q}} \mu$ if, when compared to μ:

- No diversity objective at any school is satisfied to a lesser extent under μ',
- There is at least one diversity objective satisfied to a greater extent under μ'.

Description

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  - A choice function for the schools which chooses sets of students lexicographically based on $>^{\underline{q}}$ and $\succ_c$

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  - A choice function for the schools which chooses sets of students lexicographically based on $>^{\underline{q}}$ and $\succ_c$
  - Use the School-Proposing Deferred Acceptance algorithm

$\mathbb{C}_c$ Choice Function

Step 0: If the number of acceptable students is lower than q_c, accept all of them.

$\succ_c$

Lucas

Laura

Hannah

Paul

Leon

Tim

Frank

$$q_c = 5$$

$$q_c^{t_1} = 2$$

$$q_c^{t_2} = 1$$

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$T = \{t_1, t_2, t_3\}$

$\mathbb{C}_c$ Choice Function

Step $0 < i \leq k$

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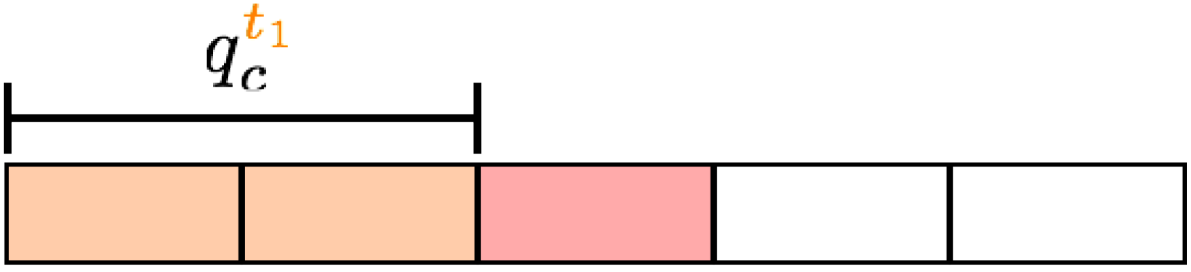

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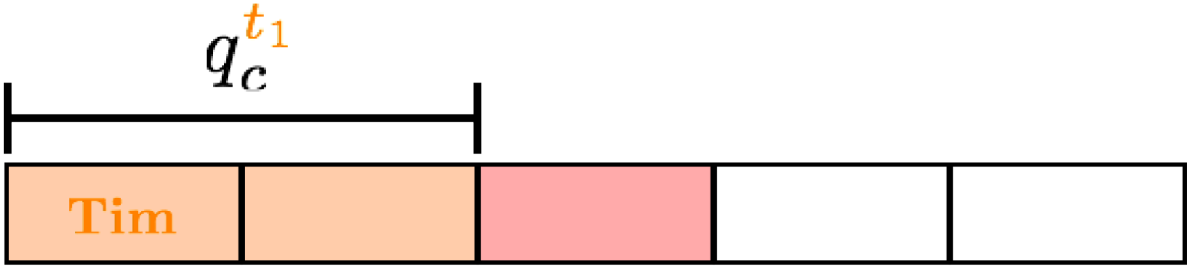

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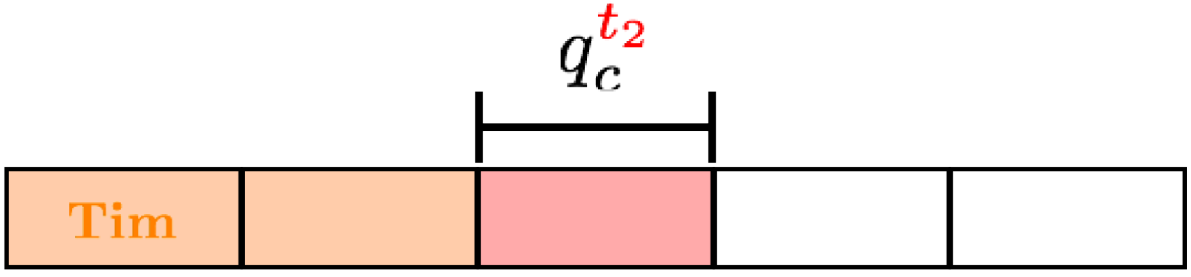

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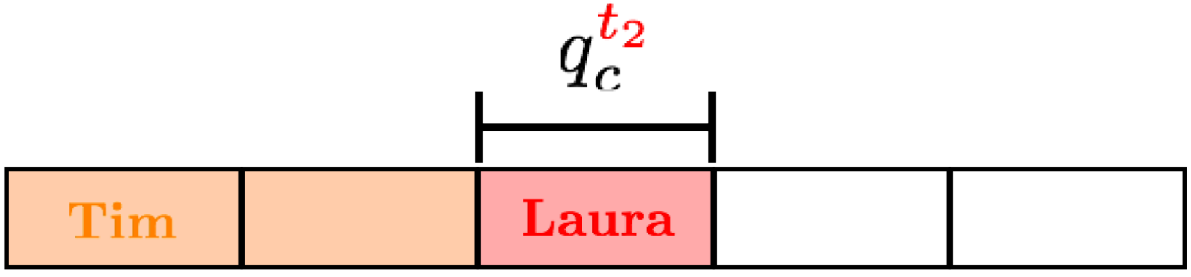

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Final Step

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Stability

Lemma

The choice function $\mathbb{C}_{\mathbf{c}}$ *satisfies the* **substitutability** *condition, that is, for all* $s, s' \in S$ *and* $S' \subseteq S$, $s \notin \mathbb{C}_{\mathbf{c}}(S' \cup \{s\}) \implies s \notin \mathbb{C}_{\mathbf{c}}(S' \cup \{s, s'\})$, *and the* **law of aggregate demand**, *that is that* $S' \subseteq S \implies |\mathbb{C}_{\mathbf{c}}(S')| \leq |\mathbb{C}_{\mathbf{c}}(S)|$. *Therefore,* $\mathbb{C}_{\mathbf{c}}$ *also satisfies* **Irrelevance of Rejected Contracts**.

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Lemma

An assignment μ *is fair with diversity if and only if* μ *is pairwise stable when schools' choice functions are* $\mathbb{C}_{\mathbf{c}}$.

Stability

Lemma (Based on Roth, 1984)

The assignment μ^C which is the outcome of the school-proposing deferred acceptance procedure is **pairwise stable** *and* **school-optimal** *in the sense that for each school c and every pairwise stable assignment μ, $\mu^C(c) = \mathbb{C}_c\left(\mu^C(c) \cup \mu(c)\right)$.*

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Theorem

The SP-Div mechanism chooses a **fair with diversity** *assignment that is undominated with respect to* $\succ^{\underline{q}}$.

▸ Proof

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Comparative Analysis

- Compare the assignments generated by the SP-Div mechanism and the Student-Proposing version of it (Hafalir et al., 2013 and Ehlers et al., 2014)

Comparative Analysis

- Compare the assignments generated by the SP-Div mechanism and the Student-Proposing version of it (Hafalir et al., 2013 and Ehlers et al., 2014)
- Assumptions: same number of students of each type, all schools are acceptable and have the same capacity, number of students equals number of seats in schools.

Preferences Domains

- **Favorite schools for each type**
  - There is a partition of schools $C = C_1 \cup \cdots \cup C_k$ where $|C_1| = \cdots = |C_k|$ and for every student $s_i \in S_i$ and $j \neq i$ it follows that $c_i \in C_i$ and $c_j \in C_j$ implies $c_i \succ_{s_i} c_j$.

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- **Tiered Schools**
  - There is a partition of schools $C = C_1 \cup \cdots \cup C_a$, where $a \geq k$, $|C_1| = \cdots = |C_a|$ and for every student $s \in S$, schools $c_i \in C_i$ and $c_{i+1} \in C_{i+1}$, student s's preferences are such that $c_i \succ_s c_{i+1}$.

Comparative Analysis - Definitions

Definition

The diversity objectives **mirror the population distribution** if the proportion of seats reserved for a type equals the proportion of students of that type in the population.

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Definition

An assignment **minimizes segregation** if the distribution of types in every school is the same as the distribution of types in the population of students.

Comparative Analysis - Results

Theorem

Every assignment generated by the SP-Div mechanism minimizes segregation when the diversity objectives mirror de population distribution.

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Every assignment generated by the SP-Div mechanism minimizes segregation when the diversity objectives mirror de population distribution.

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Proposition

Every assignment generated by the student-proposing mechanism maximizes segregation when students' preference are in the domain favorite schools for each type.

▸ Proof

Comparative Analysis - Results

Proposition

Let μ be an assignment generated by the student-proposing mechanism when the diversity objectives mirror the population distribution, and let $C^* \subseteq C$ be the set of schools such that if $c^* \in C^*$, $\mu(c^*)$ enables diversity in c^*. Then in the **tiered schools** *domain.* $\frac{|C^*|}{|C|} \geq \frac{a-1}{a}$.

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Question: can the lower-bound be binding?

Proposition 6

$S^{t_1}(S) = \{s_1, s_2, s_3, s_4\}$
$S^{t_2}(S) = \{s_5, s_6, s_7, s_8\}$

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$\succ_{s_5}$: c_1 c_2 c_3 c_4
$\succ_{s_6}$: c_2 c_1 c_3 c_4
$\succ_{s_7}$: c_1 c_2 c_4 c_3
$\succ_{s_8}$: c_2 c_1 c_4 c_3

$C = \{c_1, c_2, c_3, c_4\}$

$\succ_{c_1}$: s_1 s_2 s_3 s_4 s_5 s_6 s_7 s_8
$\succ_{c_2}$: s_1 s_2 s_3 s_4 s_5 s_6 s_7 s_8
$\succ_{c_3}$: s_1 s_2 s_3 s_4 s_5 s_6 s_7 s_8
$\succ_{c_4}$: s_1 s_2 s_3 s_4 s_5 s_6 s_7 s_8

$q_c = 2$, $q_{c_j}^{t_i} = 1$ for all $i \in \{1, 2\}$ and $j \in \{1, 2, 3, 4\}$.

Comparative Analysis - Results

The assignment generated by the student-proposing mechanism is μ, as follows:

$$\mu = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ s_1, s_5 & s_2, s_6 & s_3, s_4 & s_7, s_8 \end{pmatrix}$$

Table of Contents

1 Introduction

2 Model

3 Comparative Analysis

4 Incentives

5 Conclusion

Strategy-proofness

Proposition

The DDA mechanism is not strategy-proof.

Strategy-proofness

Proposition

The DDA mechanism is not strategy-proof.

Theorem

There is no mechanism that is fair with diversity, undominated with respect to $>^{\underline{q}}$ *and strategy-proof.*

Large Markets

- To analyze incentives induced by the DDA mechanism is large markets, we use the concept of **Strategy-proofness in the Large (SP-L)**, introduced by Azevedo and Budish (2013)

Large Markets

- To analyze incentives induced by the DDA mechanism is large markets, we use the concept of **Strategy-proofness in the Large (SP-L)**, introduced by Azevedo and Budish (2013)
- A mechanism is SP-L if for any student, full-support iid distribution over other students' reports and $\varepsilon > 0$, there is a large enough market such that the student maximizes her expected utility to within ε by reporting her preferences truthfully.

Large Markets

- Mechanisms that are not SP but that are SP-L have been shown theoretically to have approximate incentives for truthtelling in large markets
  - Walrasian Mechanism
  - Gale-Shapley deferred acceptance
  - Probabilistic serial mechanism

Large Markets

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  - Walrasian Mechanism
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  - Probabilistic serial mechanism
- Mechanisms that have empirical evidence that agents misrepresent their preferences are not SP-L
  - Boston mechanism
  - Bidding points auction for course allocation

Large Markets

- Fixed number of schools (number of seats may grow)

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  - Example: higher priority for students in the schools' neighborhood.

Large Markets

- Fixed number of schools (number of seats may grow)
- Each school has an objective criterion for setting priorities over students, based on a finite number of characteristics
  - Example: higher priority for students in the schools' neighborhood.
- Ties are broken following a lottery number

Large Markets

Definition (Azevedo and Budish, 2013)

A direct semi-anonymous mechanism $\left\{(\Psi^n)_{\mathbb{N}}, (G_h)_{h\in H}\right\}$ is **envy-free but for tie-breaking** if for each n there exists a function $x^n : (G \times [0,1])^n \to \Delta(X_0^n)$, symmetric over its coordinates, such that:

$$\Psi^n(g) = \int_{l\in[0,1]^n} x^n\left((g_i, \ell_i)_{i\in S^n}\right) dl$$

and, for all s,s',n,g and ℓ, if $\ell_s \geq \ell_{s'}$ and if g_s and $g_{s'}$ belong to the same subgroup, then

$$u_{g_s}\left[x_s^n\left((g_i, \ell_i)_{i\in S^n}\right)\right] \geq u_{g_s}\left[x_{s'}^n\left((g_i, \ell_i)_{i\in S^n}\right)\right]$$

Large Markets

Proposition (Azevedo and Budish, 2013)

If a semi-anonymous mechanism is **envy-free but for tie-breaking**, *then it is* **strategy-proof in the large**.

Theorem

The school choice with diversity mechanism SP-Div where the tie-breaking lottery $\ell \in [0,1]^n$ is drawn randomly from the uniform distribution is strategy-proof in the large.

Table of Contents

1 Introduction

2 Model

3 Comparative Analysis

4 Incentives

5 Conclusion

Conclusions

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- Affirmative action mechanisms, which typically give higher priority for students of "minority" types, may deliver completely segregated assignments.
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Conclusions

- School districts may have objectives regarding how students are distributed by types across schools.
- Affirmative action mechanisms, which typically give higher priority for students of "minority" types, may deliver completely segregated assignments.
- This paper shows that by using the school proposing deferred acceptance mechanism associated with a choice function designed for that purpose, the SP-Div mechanism has a higher ability to achieve those objectives while still yielding fair assignments.
- Incentive concerns typically raised over the use of the school proposing deferred acceptance are less of a concern in large markets.

Thanks!

Proof of Theorem 1

Theorem

The DDA mechanism chooses a fair with diversity assignment that is undominated with respect to $>^q$.

Proof.

- Suppose not. Then there is a school c, a type t and fair with diversity (pairwise stable) assignment μ^* such that $|S^t(\mu^*(c))| > |S^t(\mu^{DDA}(c))| < q_c^t$.
- But then $|S^t(\mu^*(c)) \cup S^t(\mu^{DDA}(c))| > |S^t(\mu^{DDA}(c))|$ and in the step associated with t in $\mathbb{C}_{\mathbf{c}}(\mu^*(c) \cup \mu^{DDA}(c))$, some student in $S^t(\mu^*(c)) \backslash S^t(\mu^{DDA}(c))$ is accepted.
- Contradiction with lemma based on Roth 1984.

□

Proof of Theorem 2

Proof.

When diversity objectives mirror the population distribution, each school has the objective of accepting n_1 students of each type.

- If diversity is not minimized, there is at least one school c and a type t_i such that $|S^{t_i}(\mu(c))| < n_1$.
- At some step ℓ in the deferred acceptance the set of students of type t_i that rejected school c has more than all but n_1 students. WLG let ℓ be the earliest step at which this happens to some school for any type.
- By Roth (1984), offers made by a school during the DA remain open. Since all schools are acceptable, a student that received some offer will not be left unassigned at the end of the process. Therefore, those students who rejected c are assigned to other schools.

Proof of Theorem 2

- But then at least one school proposed to more than n_1 students of type t_i.
- This can only happen if that school has less than n_1 students of some other type. Therefore there is a type t_j and a step $k < \ell$ at which that school was rejected by more than all but n_1 students of type t_j.
- Contradiction with ℓ being the earliest step.

Back

Proof of Proposition 5

Proof.

I'll show that every student $s_i \in S_i$ is accepted by some school in C_i, which is sufficient for the proof.

- Suppose that there is a student s that is not assigned to a school in C_i. Then s was rejected by all schools in C_i.
- Since every student is acceptable, s can only be rejected by a school that is already accepting q_c students.
- Since $\mathbb{C}_{\mathbf{c}}$ satisfies law of aggregate demand, by the end of each step the number of accepted students in each school never decreases. Thus by the end of step 1, at least q_c students in S_i were accepted by schools in C_i. By the end of step 2, at least $2q_c$ students were accepted in schools in C_i, etc.

Proof of Proposition 5

- By the end of step $|C_i|$ at least $q_c|C_i|$ students were accepted by schools in C_i.
- During the first $|C_i|$ steps, a student $s_j \in S_j$ could only have pointed to schools in C_j. Therefore there are at least $|C_i|q_c + 1$ students in S_i, which is a contradiction with that number being $|C_i|q_c$

□

▸ Back