College Admissions with Entrance Exams: Centralized versus Decentralized

Isa Hafalir[1] Rustamdjan Hakimov[2]

Dorothea Kübler[2] Morimitsu Kurino[3]

[1]Carnegie Mellon University

[2]WZB Berlin Social Science Center

[3]University of Tsukuba

November 7, MiP, Lisbon

Introduction

We study a **college admissions problem** in which

- colleges accept students by ranking students' efforts or scores in entrance exams, and
- students hold private information regarding their ability level that affects the cost of efforts.

Introduction

We study a **college admissions problem** in which

- colleges accept students by ranking students' efforts or scores in entrance exams, and
- students hold private information regarding their ability level that affects the cost of efforts.

Two mechanisms:

1. **Centralized college admissions (CCA)**
   - Admission decision is through a clearinghouse.
2. **Decentralized college admissions (DCA)**
   - Admission decision is independently made by each college among its applicants.

Motivation

- Every year, millions of prospective college students apply for admissions to colleges.
  - Admissions are mostly based on the results of exam scores.
  - Students put a lot of effort to do well in the exams, and are heterogeneous in terms of ability.

1. In some countries, the process is very centralized.
   - China, Greece, Korea, Taiwan, Turkey
2. In some other countries the process is decentralized.
   - Japan: For each student, only one application is made to public universities.
3. United States falls in between the two extreme cases.

The Model: $(\mathcal{C}, S, (q_1, q_2), (v_1, v_2), F)$

1. $\mathcal{C} = \{1, 2\}$: set of colleges.
   - There are two colleges: college 1 (bad) and college 2 (good).

The Model: $(\mathcal{C}, S, (q_1, q_2), (v_1, v_2), F)$

1. $\mathcal{C} = \{1, 2\}$: set of colleges.
   - There are two colleges: college 1 (bad) and college 2 (good).
2. S: a finite set of students.

The Model: $(\mathcal{C}, S, (q_1, q_2), (v_1, v_2), F)$

1. $\mathcal{C} = \{1, 2\}$: set of colleges.
   - There are two colleges: college 1 (bad) and college 2 (good).
2. S: a finite set of students.
3. (q_1, q_2): $q_C \geq 1$ is the capacity of college $C \in \mathcal{C}$.
   - Assume $q_1 + q_2 \leq |S|$.

The Model: $(\mathcal{C}, S, (q_1, q_2), (v_1, v_2), F)$

1. $\mathcal{C} = \{1, 2\}$: set of colleges.
   - There are two colleges: college 1 (bad) and college 2 (good).
2. S: a finite set of students.
3. (q_1, q_2): $q_C \geq 1$ is the capacity of college $C \in \mathcal{C}$.
   - Assume $q_1 + q_2 \leq |S|$.
4. (v_1, v_2): v_C is the cardinal utility of each student from college $C \in \mathcal{C}$.
   - $v_2 > v_1 > 0$ (attending no college).

The Model: $(\mathcal{C}, S, (q_1, q_2), (v_1, v_2), F)$

1. $\mathcal{C} = \{1, 2\}$: set of colleges.
   - There are two colleges: college 1 (bad) and college 2 (good).
2. S: a finite set of students.
3. (q_1, q_2): $q_C \geq 1$ is the capacity of college $C \in \mathcal{C}$.
   - Assume $q_1 + q_2 \leq |S|$.
4. (v_1, v_2): v_C is the cardinal utility of each student from college $C \in \mathcal{C}$.
   - $v_2 > v_1 > 0$ (attending no college).
5. F: Each student's ability is drawn i.i.d. from $[0, 1]$ according to the distribution F.
   - Each student's ability is private information.

The Model: $(\mathcal{C}, S, (q_1, q_2), (v_1, v_2), F)$

1. $\mathcal{C} = \{1, 2\}$: set of colleges.
   - There are two colleges: college 1 (bad) and college 2 (good).
2. S: a finite set of students.
3. (q_1, q_2): $q_C \geq 1$ is the capacity of college $C \in \mathcal{C}$.
   - Assume $q_1 + q_2 \leq |S|$.
4. (v_1, v_2): v_C is the cardinal utility of each student from college $C \in \mathcal{C}$.
   - $v_2 > v_1 > 0$ (attending no college).
5. F: Each student's ability is drawn i.i.d. from $[0, 1]$ according to the distribution F.
   - Each student's ability is private information.
6. Type a's utility from making effort $e =$
$$\begin{cases} v_C - \frac{e}{a} & \text{if she is assigned to college } C, \\ -\frac{e}{a} & \text{if she is assigned to no college.} \end{cases}$$

Centralized College Admissions (CCA)

The game under CCA

- Each student s simultaneously makes an effort e_s.
- Given efforts $(e_s)_{s \in S}$ of students,
  - Students with the top q_2 efforts are assigned to college 2,
  - Students with the top $(q_2 + 1)$ to $(q_1 + q_2)$ are assigned to college 1,
  - The rest of the students are not assigned to any college.

Centralized College Admissions (CCA)

The game under CCA

- Each student s simultaneously makes an effort e_s.
- Given efforts $(e_s)_{s \in S}$ of students,
  - Students with the top q_2 efforts are assigned to college 2,
  - Students with the top $(q_2 + 1)$ to $(q_1 + q_2)$ are assigned to college 1,
  - The rest of the students are not assigned to any college.

Remark

Note that this is a contest (or an all-pay auction) under incomplete information (Moldovanu and Sela, AER, 2001).

Equilibrium under CCA

Proposition 1 (Equilibrium in CCA)

In the centralized college admission mechanism, there is a unique symmetric equilibrium β^C such that for each $a \in [0,1]$,
$\beta^C(a) = \int_0^a x \left\{ f_{n-q_2,n-1}(x)\, v_2 + \left(f_{n-q_1-q_2,n-1}(x) - f_{n-q_2,n-1}(x) \right) v_1 \right\} dx.$

Decentralized College Admissions (DCA)

The Game under DCA

- Each student s chooses one college C_s and an effort e_s simultaneously.
- Given college choices $(C_s)_{s \in S}$ and efforts $(e_s)_{s \in S}$ of students,
  - Each college C admits students with the top q_C effort levels among its set of applicants ($\{s \in S \mid C_s = C\}$).
  - The rest of the students are not assigned to any college.

Decentralized College Admissions (DCA)

The Game under DCA

- Each student s chooses one college C_s and an effort e_s simultaneously.
- Given college choices $(C_s)_{s \in S}$ and efforts $(e_s)_{s \in S}$ of students,
  - Each college C admits students with the top q_C effort levels among its set of applicants $(\{s \in S \mid C_s = C\})$.
  - The rest of the students are not assigned to any college.

Remark

This is the first model of competing contests where the players sort themselves into different contests.

Equilibrium under DCA (1)

Type of Symmetric Strategy for Equilibrium

Solve for a symmetric Bayesian Nash equilibrium $(\gamma, \beta^D; c)$ where c is a cutoff,

1. A student with low ability $a \in [0, c]$
   - chooses college 1 with probability $\gamma(a)$,
   - makes effort $\beta^D(a)$.
2. A student with high ability $a \in [c, 1]$,
   - chooses college 2 for sure,
   - makes effort $\beta^D(a)$.

Comparisons

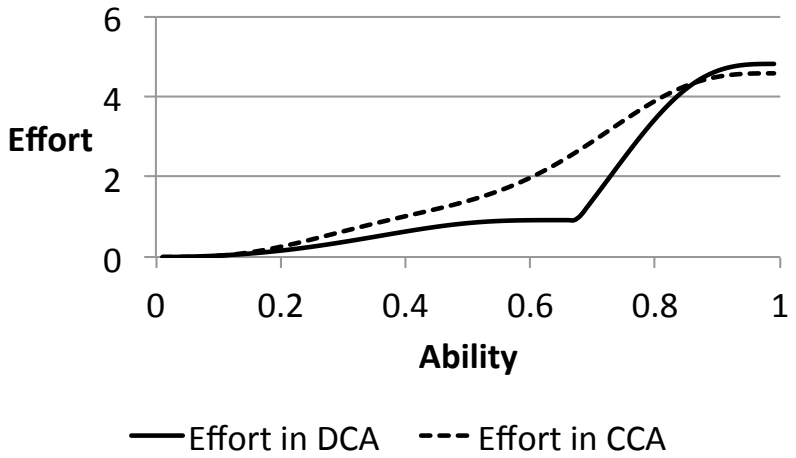
Proposition 2

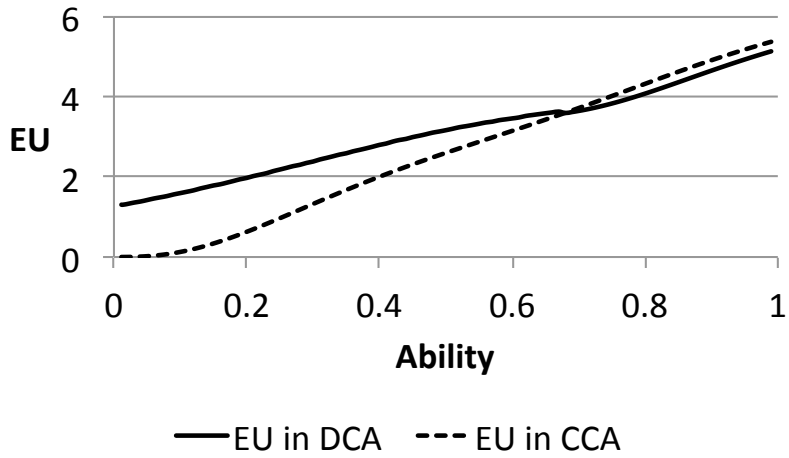
Low ability students prefer DCA to CCA if and only if $n > q_1 + q_2$.

Proposition 3 (Single crossing property)

Let c be the equilibrium cutoff.

1. If $EU^C(a) \geq EU^D(a)$ for some $a > c$, then $EU^C(a') > EU^D(a')$ for all $a' > a$, and
2. if $EU^C(a) < EU^D(a)$ for some $a > c$, then $\frac{d}{da}EU^C(a) > \frac{d}{da}EU^D(a)$.





Design of the Experiment (1)

- 2 colleges
- 12 students
- The distribution F is uniform on the ability space $[0, 100]$.
- Students received an endowment of 2,200 points. Exchange rate was 0.5 cents in Euro per point.

Design of the Experiment (1)

- 2 colleges
- 12 students
- The distribution F is uniform on the ability space $[0, 100]$.
- Students received an endowment of 2,200 points. Exchange rate was 0.5 cents in Euro per point.

here

- Five different markets

	q_2	q_1	v_2	v_1	Theoretical comparison	
					Pred. utility is higher	Pred. effort is higher
M 1	6	6	2000	1000	CCA	depends
M 2	2	2	2000	1000	DCA	no diff. in exp
M 3	2	8	2000	1000	depends; DCA in exp	CCA
M 4	3	9	2000	1800	CCA	DCA
M 5	9	1	2000	1000	no diff. in exp	no diff. in exp

Design of the Experiment (2)

- Between-subjects design.
  - Each ession with 24 subjects was either assigned to the treatment of CCA or DCA.
  - 24 subjects were split into two matching groups for the entire session.
  - Subjects played 15 rounds with one market per round by blocks of five markets.
  - Each of the five markets was played three times by every subject.
  - Abilities were drawn anew in every round.

Design of the Experiment (2)

- Between-subjects design.
  - Each ession with 24 subjects was either assigned to the treatment of CCA or DCA.
  - 24 subjects were split into two matching groups for the entire session.
  - Subjects played 15 rounds with one market per round by blocks of five markets.
  - Each of the five markets was played three times by every subject.
  - Abilities were drawn anew in every round.
- 3 sessions, i.e., 6 independent groups, for each of CCA and DCA.
- Experimental economics lab at Technical University Berlin.
- Use *z*-tree (Fischbacher, 2007).
- Sessions lasted about 90 minutes; average earnings per subject were EUR 14.10.

Treatment Comparison: Expected Utility

Result 1

- In markets 1 and 4, where $n = q_1 + q_2$, the average utility of all students in CCA is significantly higher than in DCA.
- In markets 2 and 3, the average utility of all students in DCA is not higher than in CCA, in contrast to the theoretical prediction.
- In market 5, there is no significant difference both in theory and the data.

Treatment Comparison: Expected Utility

Result 1

- In markets 1 and 4, where $n = q_1 + q_2$, the average utility of all students in CCA is significantly higher than in DCA.
- In markets 2 and 3, the average utility of all students in DCA is not higher than in CCA, in contrast to the theoretical prediction.
- In market 5, there is no significant difference both in theory and the data.

Market	q_2	q_1	v_2	v_1	Average Utility		
					predicted for realized types	CCA	DCA
1	6	6	2000	1000	CCA is higher	1223	1021
2	2	2	2000	1000	DCA is higher	111	86
3	2	8	2000	1000	DCA is higher	603	576
4	3	9	2000	1800	CCA is higher	1058	747
5	9	1	2000	1000	no difference	1183	1160

Treatment Comparison: Efforts

Result 2

- In markets 1 and 4, where $n = q_1 + q_2$, the average effort level of all students in DCA is significantly higher than in CCA.
- In market 3, the average effort level of all students in CCA is not significantly higher than in DCA.
- In markets 2 and 5, there is no significant difference in effort both in theory and in the data.

Treatment Comparison: Efforts

Result 2

- In markets 1 and 4, where $n = q_1 + q_2$, the average effort level of all students in DCA is significantly higher than in CCA.
- In market 3, the average effort level of all students in CCA is not significantly higher than in DCA.
- In markets 2 and 5, there is no significant difference in effort both in theory and in the data.

Market	q_2	q_1	v_2	v_1	Average Effort		
					predicted for realized types	CCA	DCA
1	6	6	2000	1000	DCA is higher	276	362
2	2	2	2000	1000	no difference	389	410
3	2	8	2000	2000	CCA is higher	397	354
4	3	9	2000	1800	DCA is higher	191	340
5	9	1	2000	1000	no difference	400	395

Treatment Comparison: Expected Utility of Low- and High-Ability Students

Result 3

Consider markets with $q_1 + q_2 < n$.

- In markets 2 and 3
  - the average utility of low-ability students is significantly higher in DCA as in Proposition 2,
  - the average utility of high-ability students is significantly higher in CCA as in Proposition 3.
- In market 5, there is no difference.

Table 4: Utility differences across ability quantiles

Variable	**Coefficient** (Std. Err.)
10% ability quantiles	49.008***
	(8.069)
1st quantile in DCA	98.812
	(83.255)
2nd quantile in DCA	294.889***
	(76.675)
3rd quantile in DCA	234.895***
	(73.484)
4th quantile in DCA	57.848
	(86.449)
5th quantile in DCA	-79.696
	(93.920)
6th quantile in DCA	-60.945
	(92.340)
7th quantile in DCA	-278.143***
	(91.047)
8th quantile in DCA	-103.370
	(112.019)
9th quantile in DCA	-190.702
	(118.914)
10th quantile in DCA	-186.753**
	(110.123)
Intercept	80.770*
	(45.231)
N	864
R^2	0.047
$F_{(11,852)}$	3.819

Point Predictions Regarding Individual Behavior: Effort

Result 4

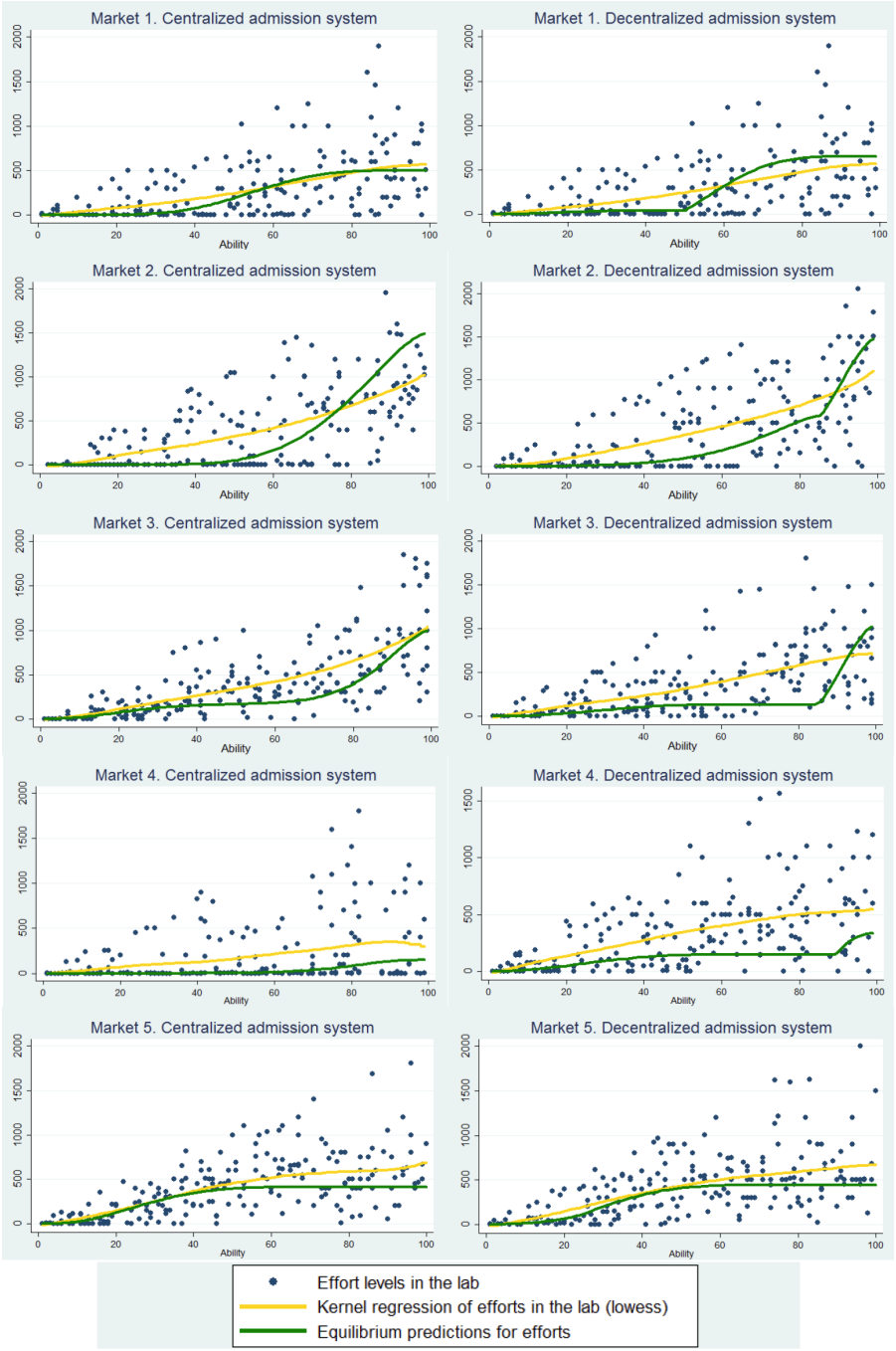
The tendency for overexertion of efforts can be observed. However, the equilibrium efforts have significant predictive power for the effort chosen.

Point Predictions Regarding Individual Behavior: Effort

Result 4

The tendency for overexertion of efforts can be observed. However, the equilibrium efforts have significant predictive power for the effort chosen.

		Mean efforts			p-value
		lab		equilibrium	
CCA	Market 1	276	>	230	0.15
	Market 2	389	>	364	0.52
	Market 3	397	>	280	0.06
	Market 4	191	>	35	0.00
	Market 5	400	>	305	0.06
DCA	Market 1	362	>	262	0.03
	Market 2	410	>	309	0.00
	Market 3	354	>	195	0.08
	Market 4	340	>	125	0.00
	Market 5	395	>	307	0.00


Market 1. Centralized admission system
Market 1. Decentralized admission system
Market 2. Centralized admission system
Market 2. Decentralized admission system
Market 3. Centralized admission system
Market 3. Decentralized admission system
Market 4. Centralized admission system
Market 4. Decentralized admission system
Market 5. Centralized admission system
Market 5. Decentralized admission system
Ability
Effort levels in the lab
Kernel regression of efforts in the lab (lowess)
Equilibrium predictions for efforts

Point Predictions Regarding Individual Behavior: Choice of Colleges in DCA

Result 5

Students above the equilibrium cutoff choose college 2 significantly more often than students below the cutoff. Across all markets, the equilibrium predictions regarding the probability of choosing the good college have siginificant predictive power for the subjects' choices.

Point Predictions Regarding Individual Behavior: Choice of Colleges in DCA

Result 5

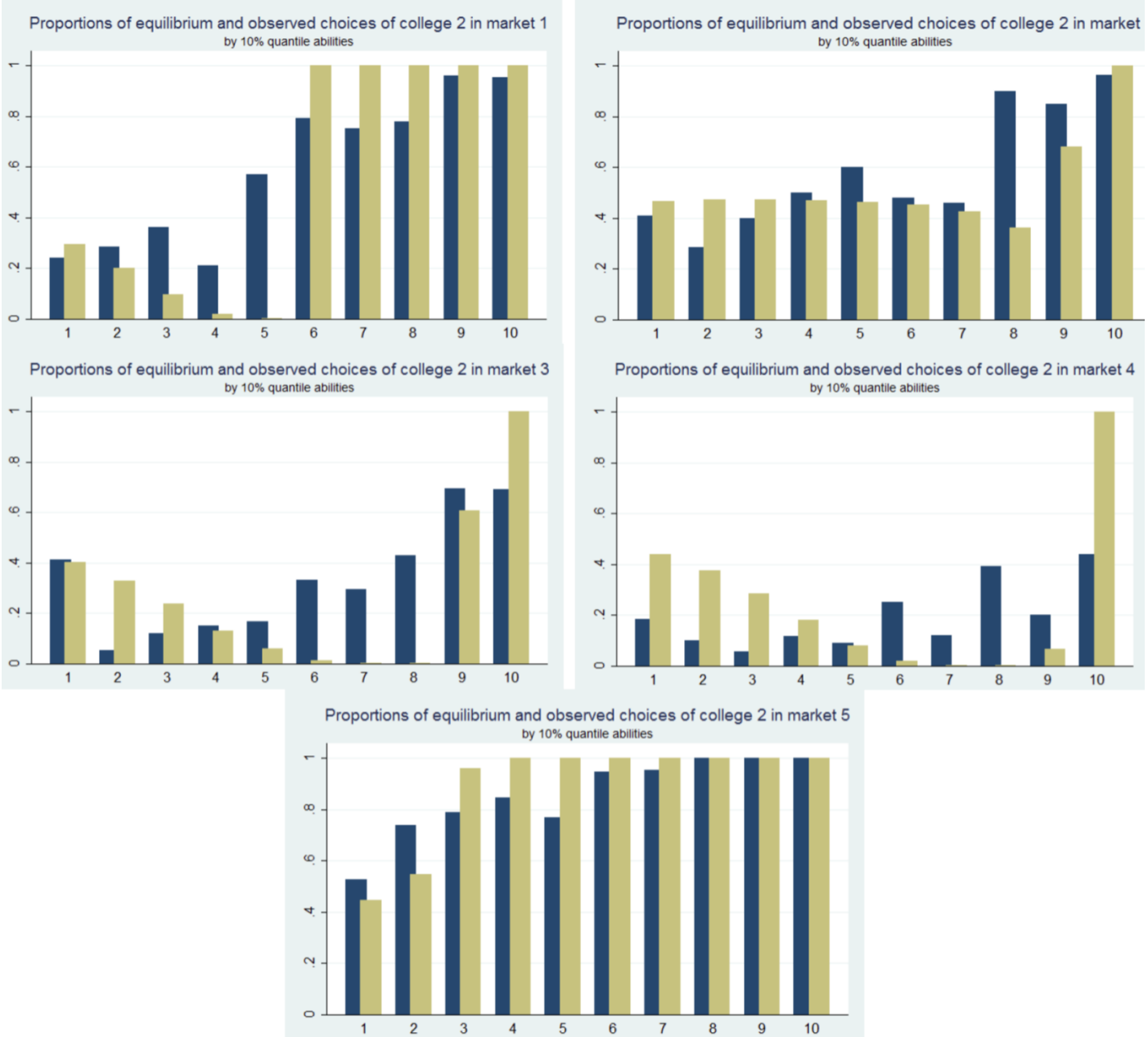
Students above the equilibrium cutoff choose college 2 significantly more often than students below the cutoff. Across all markets, the equilibrium predictions regarding the probability of choosing the good college have siginificant predictive power for the subjects' choices.

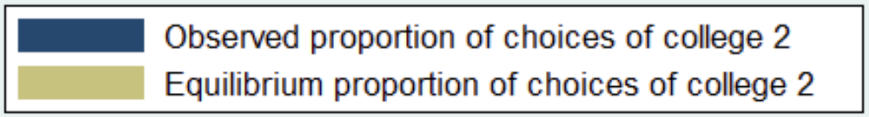
	Cutoff	Proportions of choosing the best college,depending on the cutoff.		
		Equilibrium. Below	Observed. Below	Observed. Above
Market 1	50	13%	33%	85%
Market 2	85.5	43%	51%	92%
Market 3	85.5	15%	27%	68%
Market 4	89.5	16%	17%	42%
Market 5	23.5	51%	64%	91%

Point Predictions Regarding Individual Behavior: Choice of Colleges in DCA

Table 8: Choice of the good college 2 in DCA

Variable	**Coefficient** (Std. Err.)
Equilibrium probability of choosing the good college	1.684***
	(0.106)
Intercept	-0.79***
	(0.079)
N	1080
Pseudo R^2	0.177

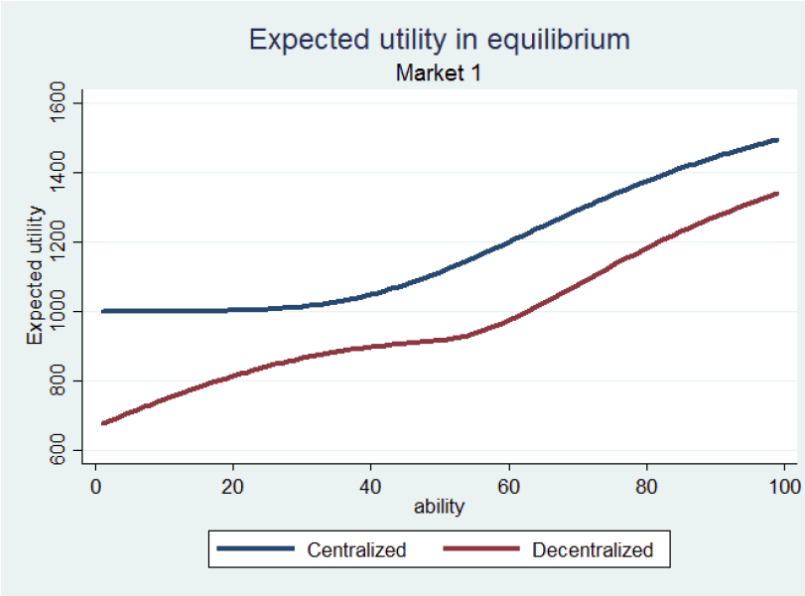

Proportions of equilibrium and observed choices of college 2 in market 1
by 10% quantile abilities
Proportions of equilibrium and observed choices of college 2 in market
by 10% quantile abilities
Proportions of equilibrium and observed choices of college 2 in market 3
by 10% quantile abilities
Proportions of equilibrium and observed choices of college 2 in market 4
by 10% quantile abilities
Proportions of equilibrium and observed choices of college 2 in market 5
by 10% quantile abilities
0
.2
.4
.6
.8
1
1
2
3
4
5
6
7
8
9
10

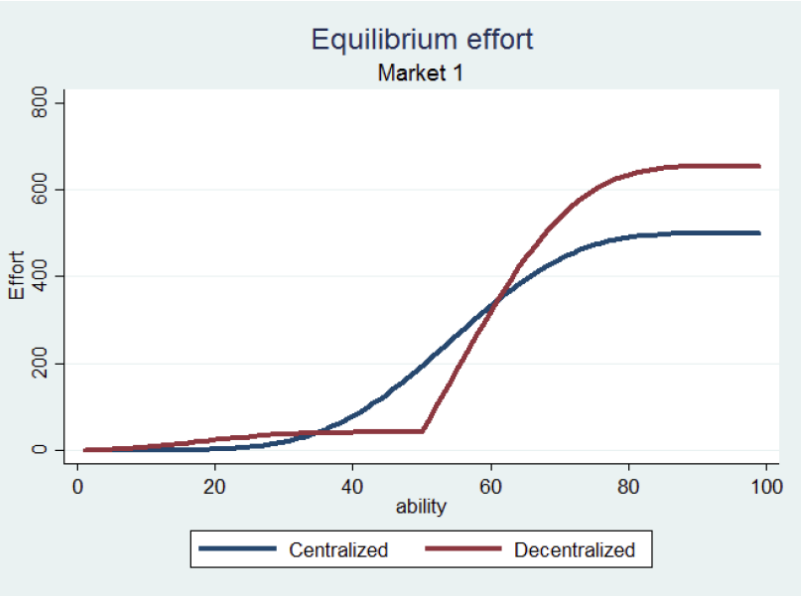

Observed proportion of choices of college 2
Equilibrium proportion of choices of college 2

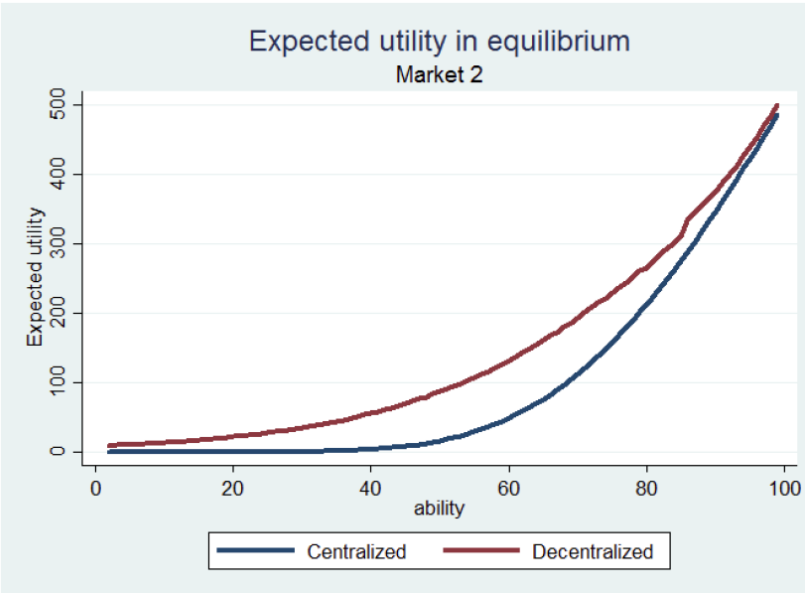
Conclusion

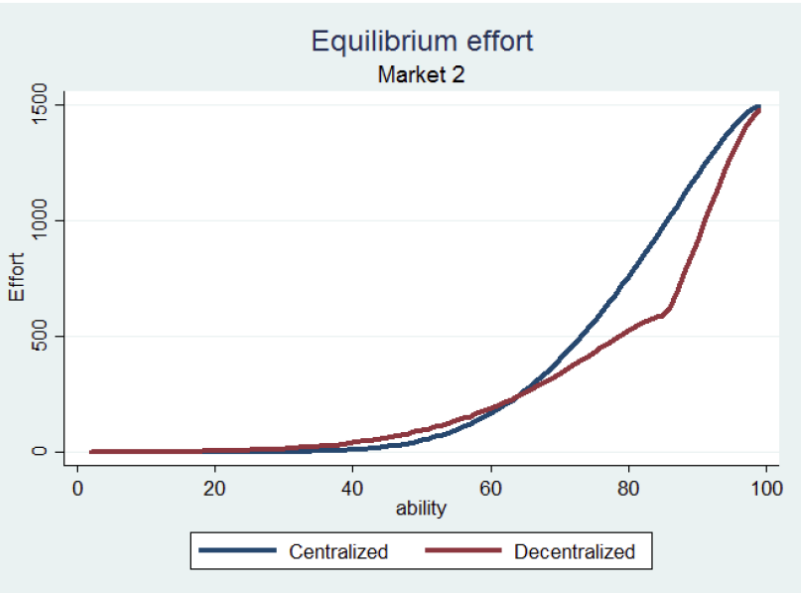
- We theoretically and experimentally study the college admissions process in which
  1. colleges accept students by considering students' exam scores,
  2. students have differing abilities which are their private information, and costs of effort for the exams are inversely related to ability levels.
- We focus on the two extreme policies:
  1. the centralized college admissions (CCA): students can freely and costlessly apply to all colleges,
  2. the decentralized college admissions (DCA): students can only apply to one college.
- Theory and experiment show consistently that:
  - low-ability students prefer DCA, and high-ability students prefer CCA.
- Theory and experiment differ with respect to:
  - overexertion of efforts in experiments, especially for DCA.

Equilibrium Predictions for Markets 1 and 2

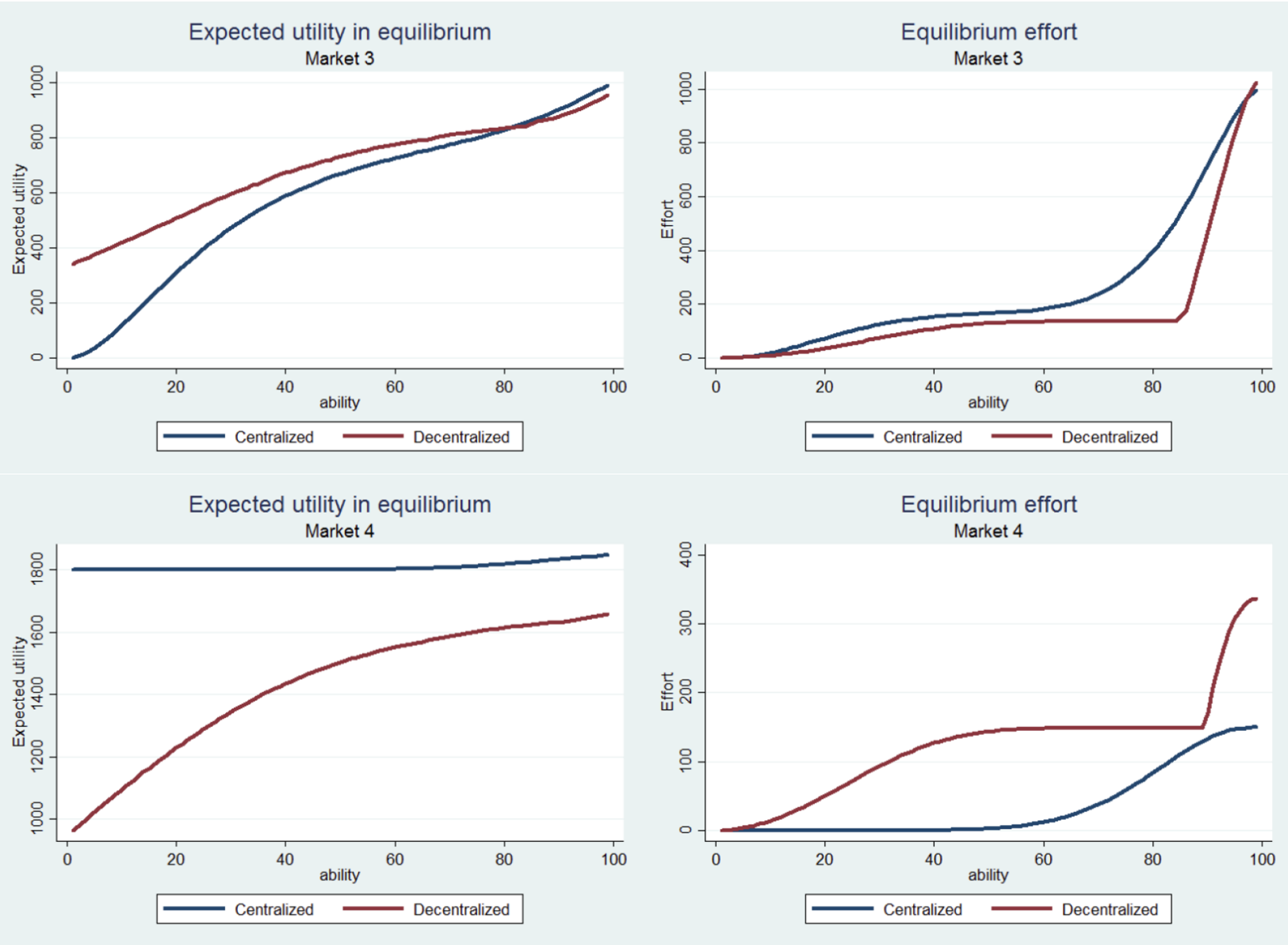




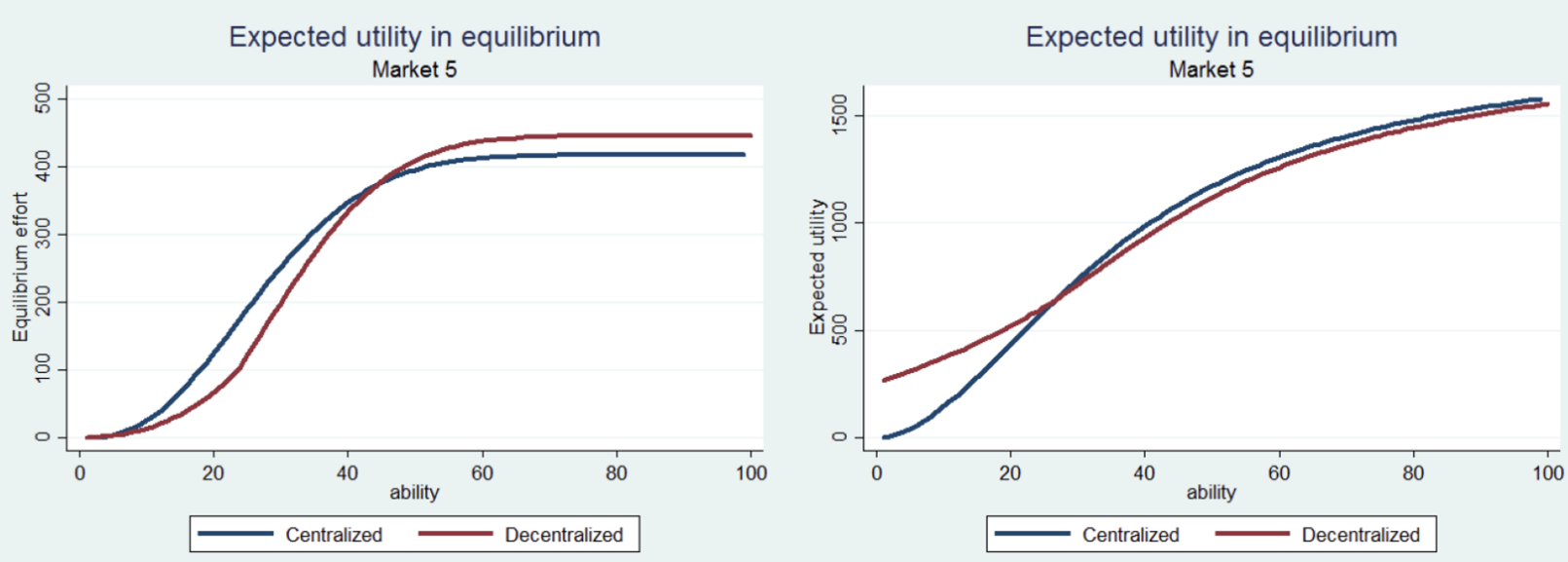




Equilibrium Predictions fo rMarkets 3 and 4



Equilibrium Predictions for Marke 5



Back to here