Matching with Waiting Times: The German Entry-Level Labor Market for Lawyers

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Motivation

- New application of matching with contracts
  - Waiting time/time of allocation as contract term
  - Multi-period setting with "false dynamics"
- Allocation of lawyers to courts for their post-graduate trainee-ship in Germany
  - After university studies, aspiring lawyers take the first state exam (bar-state)
  - Based on this grade *and* waiting time and other personal characteristics they are allocated to trainee-ship positions at regional courts
  - Often excess demand: lawyers without a matching have to wait
- Main problem
  - Lawyers can express only court but not court-time preferences
  - Time of allocation determined without using preferences
  - No coordination across *Bundesland*

Overview

- Analysing *Berlin Mechanism*
  - Not a direct mechanism
  - Neither fair nor does it respect improvements
- Propose an algorithm, with choice functions for courts and lawyers
  - Unilateral substitutes condition fails, we can use results on slot-specific choice functions as in Kominers and Sönmez (2013)
  - Under weak impatience, lawyer stable optimal allocation exists
- Time-specific lawyer offering mechanism is
  - strategy-proof, fair and respects improvements
- Relax time-specific capacity constraints by aggregating over time

Literature

- **Matching with contracts** Hatfield and Milgrom (2005); Hatfield and Kojima (2009, 2010); Aygün and Sönmez (2012); Kominers and Sönmez (2013); military application (Sönmez, 2013; Sönmez and Switzer, 2013). Our choice functions satisfy only bilateral substitutes.
- **Dynamic matching markets** Repeated one-to-one matching (Damiano and Lam, 2005; Kurino, 2009); Bloch and Houy (2012), dynamic house allocation problem (Kurino, 2014; Abdulkadiroğlu and Loertscher, 2007); Kennes et al. (Forthcoming). We apply matching with contracts and every lawyer appears just once.
- **Legal clerkships in US** Avery et al. (2007). Germany has a mandatory, regulated system.
- **Airport Landing Slots** Schummer and Vohra (2013); Schummer and Abizada (2013). We have more than one court in multiple periods.

Legal Education in Germany

- Undergraduate studies in law at university (ca. 4 years)
  - First State Exam set by each *Bundesland*
- *Referendariat* (or legal clerkship) for 2 years, partially at a court
  - Necessary to practice lawyer and employment by state agencies
  - Second State Exam
- Allocated positions by *Oberlandesgericht* to *Landgericht*
  - No coordination across *Bundesländer* (separate procedures)
  - Some coordination with *Bundesländer* with several OLGs
  - Around 8,000 positions per year

Market for Legal Clerkships in a typical *Bundesland*

- Apply to *Oberlandesgericht* for positions at *Landgericht*
  - Can rank up to 3 *Landgerichte*
- Waiting time and grade determine when allocated
- Priorities at each court depend on social criteria
  - marital status, children, existing residence, undergrad. university
- Allocation to courts within period not explicitly stated
  - Contacts suggest some form of DA used (without awareness of that fact)
- According to www.juristenkoffer.de:
  - Avg. waiting time of 0 in 8 *Bundesländer*
  - Berlin: 7-13 months; Hamburg 24 months; Northrhine-Westfalia: 2-4 months

Model

1. a finite set of lawyers $I = \{i_1, ..., i_n\}$
2. a finite set of courts $C = \{c_1, ..., c_m\}$
3. a matrix of court capacities $q = (q_{c,t})_{c \in C, t \in T}$
4. strict lawyers' preferences $P = (P_1, ..., P_n) \in \mathcal{P}$ over $C \times T \cup \emptyset$, with R weak
5. a list of court priority rankings, $\succ = (\succ_c)_{c \in C}$ over I
   - depend on grade, period of preference submission, current period and personal characteristics like undergrad university, marriage status, children etc.

Contracts and Allocations

Contracts

A contract is a triple (i, c, t) such that lawyer i is allocated a slot in court c at time t.

Allocations

Let $X \subseteq I \times C \times T$ be the set of feasible contracts; an allocation is $Y \subseteq X$:

- (i) For any $i \in I$, $|\{y \in Y : y_I = i\}| \in \{0, 1\}$.
- (ii) For any $c \in C$ and $t \in T$, $|\{y \in Y : y_C = c \cap y_T = t\}| \leq q_{c,t}$

Properties of Lawyer Preferences

Definition: Time Independence

Preferences of lawyer $i \in I$ satisfy time irrelevance if for any $c, \tilde{c} \in C$ and $t, t' \in T$, $(c,t)R_i(\tilde{c},t)$ implies $(c,t')R_i(\tilde{c},t')$.

Definition: Weak Impatience

Preferences of lawyer $i \in I$ satisfy weak impatience if for any $c \in C$ and $t < t'$, then $(c,t)R_i(c,t')$

Definition: Strict Impatience

Preferences of lawyer $i \in I$ satisfy strict impatience if for any $c, \tilde{c} \in C$ and $t < t'$, then $(c,t)R_i(\tilde{c},t')$.

Properties of Allocations and Mechanisms

Direct Mechanism

A direct mechanism is a function $\psi : \mathcal{P} \to \tilde{X}$.

Definition: Early Filling

An allocation $Y \subseteq X$ satisfies early filling if there is no $t \in T$ such that there exists some $c \in C$ such that $|\{y \in Y : y_T = t, y_C = c\}| < q_{c,t}$ and there exists some $i \in I$ such that $Y_T(i) > t$.
A mechanism ψ satisfies early fillings if for all $P \in \mathcal{P}$, $\psi(P)$ satisfies early filling.

Berlin Mechanism

- Let $Q^t = \sum_{c=1}^{|C|} q_{c,t}$ be the total capacity over all courts in time t
- Let λ_G and λ_W be grade and waiting time quotas ($\lambda_G + \lambda_W = 1$)
- Lawyers report preferences over courts

Step 1. Determine set of admitted lawyers for t

a) select up to $\lambda_G Q^t$ by grade
(break ties with waiting time, age and lottery)

b) from the rest, select up to $\lambda_W Q^t$ by waiting time
(break ties with grade, age and lottery)

Step 2. Allocation of admitted lawyers in t

a) apply the deferred-acceptance algorithm ▸ DA using selected lawyers' preferences and courts' priorities

- not selected lawyers have to wait $\Longrightarrow$ waiting time + 1

Berlin Mechanism is not fair

Example 1

- Two periods $t = 1, 2$, lawyers $I = \{i_1, i_2, i_3\}$
- Two courts $C = \{c_1, c_2\}$, with $q_{c_1,1} = q_{c_2,1} = 1$ and $q_{c_1,2} = 1$, $q_{c_2,2} = 0$
- Court-side priorities (same as grades) $\succ$: $i_1 \succ_c i_2 \succ_c i_3$ for all $c \in C$
- Lawyers i report $c_1 P_i c_2$, real preferences:

$$\begin{aligned} i_1,\ i_2: &\quad (c_1,1) P_i (c_1,2) P_i (c_2,1) \\ i_3: &\quad (c_1,1) P_i (c_2,1) P_i (c_1,2) \end{aligned}$$

$$\Longrightarrow \hat{X} = \{(i_1, c_1, 1), (i_2, c_2, 1), (i_3, c_1, 2)\}$$

This is **not fair** ▸ Fairness, since for i_2: $(c_1, 2) P_{i_2} (c_2, 1)$, although $i_2 \succ_c i_3$.

Berlin Mechanism does not respect improvements

Example 2

- Set-up as in Example 1
- Priorities changed to $\overset{\circ}{\succ}$: $i_1 \overset{\circ}{\succ}_c i_3 \overset{\circ}{\succ}_c i_2$

  $\Longrightarrow \tilde{X} = \{(i_1, c_1, 1), (i_2, c_1, 2), (i_3, c_2, 1)\}$
- if i_2 improves ranking, from $\overset{\circ}{\succ}$ to $\succ$

  $\Longrightarrow \hat{X} = \{(i_1, c_1, 1), (i_2, c_2, 1), (i_3, c_1, 2)\}$
- i_2 prefers $\tilde{X}$ to $\hat{X}$

$\Longrightarrow$ Berlin Mechanism does **not respect improvements** ▸ Respect of Improvements

Properties of the Berlin Mechanism

- Under time independence, submitting court preferences consistent with true preferences is optimal

Proposition

The outcome of the Berlin Mechanism fills positions early.

Proposition

Under strict impatience and time independence the Berlin Mechanism is fair and respects improvements.

Choice Functions

- Static: arrival and preferences of all lawyers already known in period 1

Lawyer i's choice function

$$\mathbb{C}_i(Y) \equiv \max_{P^i} Y$$

Court c's time-specific choice function

$\mathbb{C}_c^{ts}(Y)$:

Step 0**:** Reject all contracts $y \in Y$ with $y_C \neq c$.

Step $1 \leq t \leq T$**:** Consider contracts $y \in Y$ with $y_T = t$. Accept one by one according to $\succ_c$ until capacity $q_{c,t}$. Then reject all other contracts y with $y_T = t$. If a contract of lawyer y_I has been accepted, reject all other y' with $y'_I = y_I$. If all contracts have been considered, end the algorithm. Otherwise move to the next step, unless $t = T$, then reject all remaining contracts.

Properties of Choice Function

Lemma 1

The static choice functions are fair. ▸ Fair choice function

Lemma 2

The static choice functions satisfy bilateral substitutes ▸ Substitutes and irrelevance of rejected contracts ▸ IRC.

$\Longrightarrow$ COP ▸ COP produces a stable allocation ▸ Stability (Theorem 1, HK 2010)

Slot-specific choice function

The time-specific choice function is a special case of the general slot-specific choice function of Kominers and Sönmez (2013) ▸ Slot-Specific.

Properties of Choice Functions

Example 3 (Failure of unilateral substitutes ▸ Substitutes)

- $x = (j, c, t)$, $Y = \{(j, c, t+1)\}$ and $z = (i, c, t+1)$,
- $j \succ_c i$ and $q_{c,t} = q_{c,t+1} = 1$

$\Longrightarrow z \notin \mathbb{C}_c^{ts}(Y \cup \{z\}) = \{(j, c, t+1)\}$, but
$\Longrightarrow z \in \mathbb{C}_c^{ts}(Y \cup \{x, z\}) = \{(j, c, t), (i, c, t+1)\}$

Example 4 (Failure of Law of Aggregate Demand ▸ LAD)

- $x = (j, c, t)$, $Y = \{(i, c, t), (j, c, t+1)\}$
- $j \succ_c i$ and $q_{c,t} = q_{c,t+1} = 1$

$\Longrightarrow \mathbb{C}_c^{ts}(Y) = Y$ and $\mathbb{C}_c^{ts}(Y \cup \{x\}) = \{x\}$
$|\mathbb{C}_c^{ts}(Y)| > |\mathbb{C}_c^{ts}(Y \cup \{x\})|$

Results

Lemma 4

Under weak impatience, COP using the static choice function $\mathbb{C}_c^{ts}$ produces the lawyer-optimal stable allocation.

Using results in Kominers and Sönmez (2013) and Sönmez (2013):

Proposition

The time-specific lawyer offering stable mechanism

- is (group) strategy-proof
- respects improvements
- is fair
- does not fill positions early

Flexible Choice Function

- What if greater budgetary freedom for courts?

Flexible Choice Function

$C_c^{flex}(Y)$:

Step 0: Reject all contracts $y \in Y$ with $y_C \neq c$

Step $1 \leq k \leq T$**:** Accept contracts one by one according to $\succ_c$ until capacity $Q_c = \sum_{t=1}^{T} q_{c,t}$. If there is more than one contract of a lawyer being considered, accept the one with a lower t. Reject all other contracts.

Results about the Flexible Choice Function

Properties of the Flexible Choice Function

The Flexible Choice Function, C_c^{flex}:

- satisfies the substitutes ▸ Substitutes condition,
- satisfies the Law of Aggregate Demand,
- satisfies irrelevance of rejected contracts,
- is fair.

COP using Flexible Choice Function

The mechanism using the COP and the flexible choice function:

- is (group) strategy-proof
- is fair and respects improvements
- produces a lawyer-optimal stable allocation

Time-Specific vs Flexible Choice Function

- Flexible choice function relaxes feasibility constraints
- Would expect flexible choice function to yield "better" results
- But: more possible blocking coalitions under flexible choice function
- Comparison depends on particular preference profiles

Optimality of flexible choice function

Suppose priorities are acyclical ▸ Acyclicality or that all lawyers have identical preferences, then the flexible lawyer-optimal stable mechanism is never Pareto dominated by the time-specific lawyer proposing mechanism.

Time-Specific vs Flexible Choice Function - Example

Example 5

$I = \{i, j, k\}$, $C = \{a, b\}$, $q_{a,1} = q_{a,2} = q_{b,1} = 1$ and $i \succ_c j \succ_c k$ for $c = a, b$

- **i:** $(a,1) P_i \emptyset$
- **j:** $(a,1) P_j (b,1) P_j (a,2)$
- **k:** $(a,1) P_k (a,2) P_k (b,1)$

Outcomes are:

- **Flexible:** $\{(i,a,1), (j,a,1), (k,b,1)\}$
- **Time-Specific:** $\{(i,a,1), (j,b,1), (k,a,2)\}$

Neither Pareto dominates the other:

- j prefers Flexible to Time-Specific
- k prefers Time-Specific to Flexible

Conclusion

- Berlin Mechanism has flaws but satisfies early filling by construction
- Proposal of a new mechanism with choice functions for lawyers and courts, and waiting time as contract term
- Use results on slot-specific choice functions of Kominers and Sönmez (2013)
- Time-specific lawyer proposing mechanism is strategy-proof, fair and respects improvements, but not early filling
- Greater budgetary flexibility not necessarily better

Thank you!

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Properties of Allocations and Mechanisms 1

Fairness

An allocation $Y \subseteq X$ is fair, if for any pair of contracts $x, y \in Y$ with $x_I \neq y_I$ and $(x_C, x_T) P_{y_I} (y_C, y_T)$, then $x_I \succ_{x_C} y_I$. A mechanism ψ is fair if its outcome $\psi(P)$ is fair for all $P \in \mathcal{P}$.

▸ Berlin Mechanism

Pareto-Efficiency

An allocation $Y \in \tilde{X}$ is Pareto efficient if there exists no allocation $\tilde{Y} \in \tilde{X}$ such that for all $i \in I$ $\tilde{Y}(i) R_i Y(i)$ and there exists at least one $i \in I$ such that $\tilde{Y}(i) P_i Y(i)$.

Strategy-Proofness

Mechanism ψ is strategy-proof if for all $i \in I$, for all $P \in \mathcal{P}$ and for all $\tilde{P}_i \in \mathcal{P}_i$ we have $\psi(P) R_i \psi(\tilde{P}_i, P_{-i})$.

Properties of Allocations and Mechanisms 2

Priority Profile Improvement

A priority profile $\succ$ is an unambiguous improvement over another priority profile $\succ'$ for lawyer i if:
- the ranking of lawyer i is at least as good under $\succ$ as $\succ'$ for any court c,
- the ranking of lawyer i is strictly better under $\succ$ than under $\succ'$ for some court c,
- the relative ranking of other lawyers remains the same between $\succ$ and $\succ'$ for any court c

Respect of Improvements

A mechanism ψ respects unambiguous improvements if a lawyer never receives a strictly worse assignment as a result of an unambiguous improvement in her court priorities.

▸ Berlin Mechanism

Properties of Choice Functions 1

Substitutes

Contracts are **substitutes** for c if there do no exist contracts $x, z \in X$ and a set of contracts $Y \subseteq X$ such that $z \notin \mathbb{C}_c(Y \cup \{z\})$ and $z \in \mathbb{C}_c(Y \cup \{x, z\})$.

Unilateral Substitutes

Contracts are **unilateral substitutes** for c if there do not exist contracts $x, z \in X$ and a set of contracts $Y \subseteq X$ such that $z_I \notin Y_I$, $z \notin \mathbb{C}_c(Y \cup \{z\})$ and $z \in \mathbb{C}_c(Y \cup \{x, z\})$.

Bilateral Substitutes

Contracts are **bilateral substitutes** for c if there do not exist contracts $x, z \in X$ and a set of contracts $Y \subseteq X$ such that $z_I, x_I \notin Y_I$, $z \notin \mathbb{C}_c(Y \cup \{z\})$ and $z \in \mathbb{C}_c(Y \cup \{x, z\})$.

◂ Choice Function 1 ◂ Choice Function 2 ◂ Choice Function 3

Properties of Choice Functions 2

Irrelevance of Rejected Contracts

Contracts satisfy irrelevance of rejected contracts for court c if for all $Y \subset X$ and for all $z \in X \setminus Y$, we have $z \notin \mathbb{C}_c(Y \cup \{z\})$ implies $\mathbb{C}_c(Y) = \mathbb{C}_c(Y \cup \{z\})$.

Law of Aggregate Demand

The preferences of court $c \in C$ satisfy the law of aggregate demand if for all $X' \subseteq X'' \subseteq X$, $|\mathbb{C}_c(X')| \leq |\mathbb{C}_c(X'')|$.

Fair Choice Function

For any court c, choice function $\mathbb{C}_c$ is fair if for any set of contracts $Y \subseteq X$, and any pair of contracts $x, y \in Y$ with $x_C = y_C = c$, $y_I \succ_c x_I$, $y_T = x_T$ and $x \in \mathbb{C}_c(Y)$ then there exists $z \in \mathbb{C}_c(Y)$ such that $z_I = y_I$.

◂ Choice Function ◂ Choice Function 2

Stability

Stability

An allocation $Y \subseteq \tilde{X}$ is stable with respect to choice functions $(\mathbb{C}_c)_{c=1}^{|C|}$ if we have:

1. individually rational: $\mathbb{C}_i(Y) = Y(i)$ and $\mathbb{C}_c = Y(c)$ for all $c \in C$ and
2. no blocking coalition: there is no court $c \in C$ and a blocking set $Y' \neq \mathbb{C}_c(Y)$ such that $Y' = \mathbb{C}_c(Y \cup Y')$ and $Y' R_i Y$ for all $i \in Y'_I$.

◂ Choice Function

Slot-specific Choice Functions

Slot-Specific Choice Function

Each court has S_c, denoted s_c^l with $l = 1, ..., Q_c$. Let Π_c^l be priority order of slot l at court c over contracts. Given a set of contracts Y:

- First, slot s_c^1 is assigned x^1 which Π_c^1-maximal in Y
- Then, slot s_c^2 is assigned x^2 which Π_c^2-maximal in $Y \setminus x \in Y : x_l = x_l^1$
- Continue this through all slots.

◂ Choice Function

Cumulative Offer Process

Cumulative Offer Process

- Step 1: One (arbitrarily chosen) bidder offers her first choice contract, x_1 to a court c. The court $\tilde{c}$ holds the contract if it is acceptable and rejects it otherwise. Let $H_{\tilde{c}}(1) = \{x_1\}$ and $H_c(1) = \emptyset$ otherwise.

In general,

- Step $m \geq 2$: One of the bidders for whom no contract is currently held by a court offers the most preferred contract, say x_m to a court $\tilde{c}$, that has not been rejected in previous steps. Court $\tilde{c}$ holds $C_{\tilde{c}}(H_{\tilde{c}}(m-1) \cup \{x_m\})$ and rejects all other contracts. Let $H_{\tilde{c}}(m) = H_{\tilde{c}}(m-1) \cup \{x_m\}$ and $H_c(m) = H_c(m-1)$.

The algorithm ends when no bidder offers another contract. The contracts held by the courts at the final step are the resulting allocation.

◂ Choice Function

Deferred-Acceptance Algorithm

Deferred-Acceptance Algorithm (Gale and Shapley, 1962)

Consider all lawyers to be allocated a slot in period t,

- **Step 1:** All lawyers make an offer to their most preferred court. Each court c puts on hold up to $q_{c,t}$ lawyers according to its priority $\succ_c$ over lawyers and rejects all others.
- **Step k:** All lawyers whose offers have been rejected in the previous round, make an offer to their most preferred court that has not rejected them. Courts put on hold up to $q_{c,t}$ lawyers, choosing among new offers and those on hold from the previous step. All other offers are rejected.

The algorithm ends when no lawyer has any offer left to make. The offers on hold by the courts in the last step are the resulting allocation.

◂ Berlin

Acyclicality

Definition from Ergin (2002)

- Let $U_c(i) = \{j \in I | j \succ_c i\}$ for all $c \in C$ and $\succ_c$

Acyclicality

A cycle consists of $a, b \in C$ and $i, j, k \in I$ such that:

- **Cycle:** $i \succ_a j \succ_a k \succ_b i$
- **Scarcity:** there is $I_a, I_b \subset I \setminus \{i, j, k\}$ such that $I_a \subset U_a(j)$, $I_b \subset U_b(i)$, $|I_a| = q_a - 1$ and $|I_b| = q_b - 1$

Priorities are acyclical if they have no cycles.

◂ Comparison