Timing and Centralized University Admissions

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I. Introduction

Introduction

- Preferences $\longrightarrow$ Mechanisms$\longrightarrow$ Allocation
  - choice of mechanisms: DA (strategy-proof), and Boston (non-strategyproof)...
  - priorities: scores/grades from entrance exams and secondary schools...

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  - choice of mechanisms: DA (strategy-proof), and Boston (non-strategyproof)...
  - priorities: scores/grades from entrance exams and secondary schools...
- In practice, mechanisms often interact with the timing to submit preferences.

Timing of Preference Submission

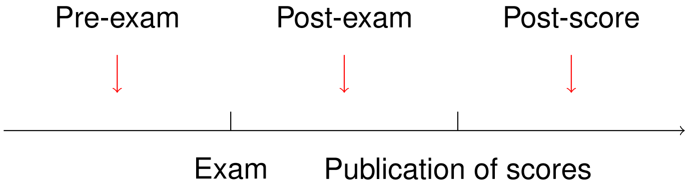

Figure: Timing to submit preferences

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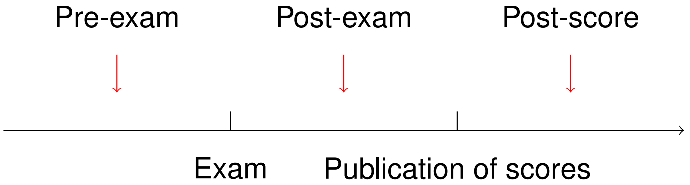

Figure: Timing to submit preferences

- Examples: Ireland (post-exam), UK (post-exam), Mexico city (pre-exam), and China (all three) etc.

This paper

- Questions: Which timing is good for students? Which timing is good for universities?
- We investigate the non-strategyproof Boston mechanism, and show when students are described by 2 dimensions, scores and preference intensity
  - students take a cutoff strategy on score (post-score), preference intensity (pre-exam), and a trade-off function between the two (post-exam),
  - pre-exam and post-exam submissions under Boston can reduce sorting.

Literature: school choice

- Complete information about priority: the set of Nash equilibrium outcomes induced by Boston is equal to the set of stable matching (under true preferences) (Ergin and Sonmez, 2006)
- Incomplete information about priority as a result of lottery: Boston is ex-ante efficient (Abdulkadiroğlu et al., 2011)

Literature: university admissions

- Early action and admission strategies adopted by universities (Avery and Levin, 2010)
- Sorting may fail when students application is costly, and college evaluation of their application are uncertain (Chade et al., 2014)
- Experimental study on German university admissions, the more pupular universities admit less good students, and the less popular universities admit better students when the sequential mechanism (non-strategy-proof) is used (Braun et al., 2014)
- Empirical analysis on one top Chinese university: the top university more likely to admit students with lower exam scores when the pre-exam Boston is used (Wu and Zhong, 2014)

II.An Illustration

Setting

- 3 universities A, B, and C; capacities are: $q_A = q_B = \frac{1}{4}$, $q_C = \frac{1}{2}$.
- Universities have strict and identical preferences.

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- Universities have strict and identical preferences.
- A unit mass of students, described by two dimensions:
  - preference intensity $y \sim U\left[\frac{1}{4}, \frac{5}{4}\right]$, a student knows her y and the distribution of y;
  - score $e \sim U[0, 1]$, depending on the timing a student may or may not know her own score, but knows the distribution of e;
  - e and y are independently distributed.

Setting

- The utilities are: $u^A = y$, $u^B = 1 - y$, and $u^C = -\frac{1}{4}$.
- Two students preference profiles:
  - $P_1 : A \succ B \succ C$ if $y \geqslant \frac{1}{2}$
  - $P_2 : B \succ A \succ C$ if $y < \frac{1}{2}$
- A strategy $\sigma : \left[\frac{1}{4}, \frac{5}{4}\right] \times [0, 1] \to \Delta(\Pi)$, where $\Delta(\Pi)$ denote the set of probability distributions over Π the rank-ordered list of universities:
  - $\sigma_1 : A \succ B \succ C$
  - $\sigma_2 : B \succ A \succ C$

Boston mechanism

Round 1: each student proposes to the university of her first choice. University j accepts definitely the students whose scores are higher than or equal to $\hat{e}_j^1$, the minimum threshold score of the first round, such that the mass of students accepted is smaller or equal to the available capacity, and reduces its capacity by the mass of students accepted. If the capacity of university j is filled, then set the final minimum acceptance threshold $\hat{e}_j = \hat{e}_j^1$.

Boston mechanism

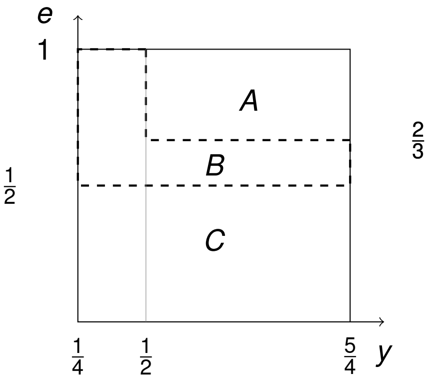
In general, at

Round r: each student who was rejected in previous round, proposes to her r-th ranked choice. If the university still has capacities, then it accepts definitely the students whose scores are higher than or equal to $\hat{e}_j^r$, the minimum threshold score of the current round, such that the mass of students accepted is smaller or equal to the available capacity. Then it reduces its capacity by the mass of students accepted in this round. If the capacity of university j is filled, then set the final minimum acceptance threshold $\hat{e}_j = \hat{e}_j^r$.

The mechanism terminates when the capacity is finished or there are no more students to be assigned.

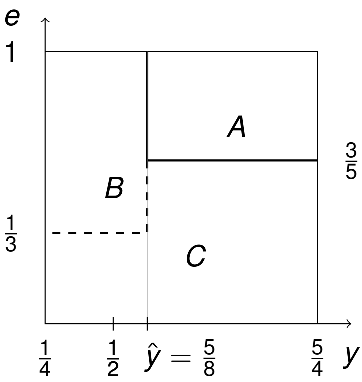
The post-score Boston mechanism

- Students observe their scores and the rank of their scores.
- At equilibrium, capacities at both A and B are filled after first round.
- Students with $y < \frac{1}{2}$ submit preference truthfully; the strategies of students with $y \geq \frac{1}{2}$ depend on their scores.



The pre-exam Boston mechanism

- Students have no information about thier scores, and the realization of scores is random.
- At equilibrium, capacities at both A and B are filled after first round.
- Students with $y < \frac{1}{2}$ submit preference truthfully; the strategies of students with $y \geq \frac{1}{2}$ depend on their preference intensity.



Comparisons

	Post-score	Pre-exam	
Min threshold at A	0.6667	0.6	$\downarrow$
Min threshold at B	0.5	0.333	$\downarrow$
Min threshold at C	0	0	
Average scores at A	0.2083	0.2	$\downarrow$
Average scores at B	1.6667	1.6667	
Average scores at C	0.125	0.1333	$\uparrow$
Welfare for students admitted to A	0.2188	0.2344	$\uparrow$
Welfare for students admitted to B	0.0956	0.1406	$\uparrow$

III.The Main Model

Setting

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- Universities have strict and identical preferences.
- A unit mass of students, described by two dimensions:
  - preference intensity $y \sim G$, a student knows her y and the distribution of y;
  - score $e \sim F$, depending on the timing a student may or may not know her own score, but knows the distribution of e;
  - e and y are independently distributed.
- The utilities are: $u^A = y$, $u^B = 1 - y$, and $u^C \leq 0$. Students are indifferent between A and B when $y^* = \frac{1}{2}$
- Students preference profiles: $P_1 : A \succ B \succ C$ if $y_i \geqslant \frac{1}{2}$ and $P_2 : B \succ A \succ C$ if $y_i < \frac{1}{2}$
- Strategies: $\sigma_1 : A \succ B \succ C$ and $\sigma_2 : B \succ A \succ C$

The post-score Boston

Lemma

Let $G(y^*) < \frac{1}{2}$ be the measure of students who prefer university B to university A ($y < y^*$). In any Nash equilibrium induced by the post-score Boston mechanism,
(1) students with preference intensity $y < y^*$ submit their preferences truthfully,
(2) students with preference intensity $y \geqslant y^*$ submit their preferences truthfully if $e \geqslant \hat{e}_A^{poss}$, and manipulate their preferences as $B \succ A \succ C$ otherwise.

The post-score Boston

Proposition

In the unique Nash equilibrium outcome induced by the post-score Boston mechanism, the minimum acceptance thresholds are equal to the minimum acceptance thresholds of the deferred acceptance mechanism under the true preferences: $\hat{e}_j^{poss} = \hat{e}_j^{DA}$ *. In addition, university A is more selective than university B.*

The pre-exam Boston

- At equilibrium, students with preference intensity $y < y^*$ submit preference truthfully,
- At equilibrium, students with preference intensity $y \geqslant y^*$ are indifferent when

$$y \cdot P_A(\sigma_1) = (1 - y) \cdot P_B(\sigma_2) \tag{1}$$

solving equation 1 gives us the cutoff $\hat{y}$.

The pre-exam Boston

Lemma

Let $G(y^*) < \frac{1}{2}$ be the measure of students who prefer university B to university A ($y < y^*$). In any Bayesian Nash equilibrium induced by the pre-exam Boston mechanism, there exists a unique cutoff $\hat{y}$ such that students submit their preferences as $A \succ B \succ C$ if $y \geqslant \hat{y}$, and submit their preferences as $B \succ A \succ C$ otherwise.

Proof.

1. Let $\phi(y) \equiv y \cdot P_A(\sigma_1) - (1 - y) \cdot P_B(\sigma_2)$, then $\phi_y(y) > 0$.
2. Uniqueness is guaranteed by fixed point theorem.

□

The pre-exam Boston

Proposition

In the unique Bayesian Nash equilibrium outcome induced by the pre-exam Boston mechanism, $\hat{e}_A^{pree} < \hat{e}_A^{poss}$, $\hat{e}_B^{pree} < \hat{e}_B^{poss}$, *and* $\hat{e}_C^{pree} = \hat{e}_C^{poss}$.

The post-exam Boston

- After exam, students receive a signal x about their score.
- The signal x and score e satisfy monotone likelihood ratio property(MLRP), such that a student who obtains a high score is more likely to receive a higher signal.
- In addition, let the posterior $f(e|x)$ satisfy Convex Distribution Function Condition (CDFC).

The post-exam Boston

- At equilibrium, students with preference intensity $y < y^*$ submit preference truthfully,
- students with preference intensity $y \geqslant y^*$ are indifferent when

$$y \cdot P_A\left(\sigma_1, e|x\right) = (1 - y) \cdot P_B(\sigma_2, e|x) \tag{2}$$

where the pair $(\hat{y}, \hat{e}|\hat{x})$ is the solution to equation 2.

The post-exam Boston

Lemma

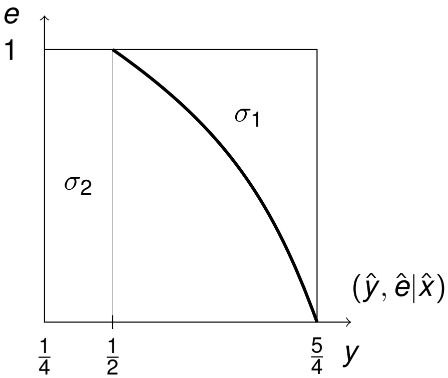
Let $G(y^*) < \frac{1}{2}$ be the measure of students who prefer university B to university A ($y < y^*$). In any Bayesian Nash equilibrium induced by the post-exam Boston mechanism, there exists a unique cutoff locus $(\hat{y}, \hat{e}|\hat{x})$ such that students submit their preferences as $A \succ B \succ C$ if the (y, x) pair is above $(\hat{y}, \hat{e}|\hat{x})$, and submit their preferences as $B \succ A \succ C$ otherwise.

Proof.

1. Cutoff form: Let $\phi(y, e|x) \equiv y \cdot P_A(\sigma_1, e|x) - (1 - y) \cdot P_B(\sigma_2, e|x)$, then $\phi_y > 0$ and $\phi_x > 0$
2. Uniqueness is guaranteed by fixed point theorem.

□

Back to the illustrative example



Comparison

Theorem

In any (Bayesian) Nash equilibrium outcome induced by the Boston mechanism, when changing from post-score submission to post-exam or pre-exam submission,
(1) both the minimum acceptance threshold and the average score at the more popular university decrease;
(2) the minimum acceptance threshold at the less popular university decreases, but the average score is non-decreasing;
(3) the minimum acceptance threshold at the least preferred university remains the same, and the average score is non-decreasing.

Comparison

Theorem

In the unique (Bayesian) Nash equilibrium outcome induced by the Boston mechanism, the average welfare of students admitted to both the popular and less popular universities under post-score submission is dominated by post-exam, and by the pre-exam submission.

III. Conclusions

Conclusions

- Boston: students take a cutoff strategy on score (post-score), preference intensity (pre-exam), and a trade-off function between both (post-exam).
- pre-exam and post-exam submission under Boston can improve students average welfare.
- pre-exam and post-exam submission under Boston can reduce sorting.
- Further research: in dynamic setting, students preference distribution endogenously determined by the minimum acceptance thresholds...

Thank you!

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