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Equitable Top Trading Cycles mechanism

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Motivation

- Since 1987, school choice programs have been growing in popularity across the US, aiming to overcome the **inequality gap** between students from rich and poor families.

- Main issues of concern:
  - fairness/equity
  - efficiency
  - incentive compatibility

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Motivation

- Equity and welfare conflict
  - Fairness: DA
  - Pareto efficiency: TTC

- San-Francisco, Denver, New Orlean put TTC in practice.

- Can we improve on equity of TTC?

School Choice Problem

- A *school choice problem*
  - *Preference profile* of students
  - *Priority orders* for schools.
  - *Quotas*
- As a result: **matching**

Properties of mechanisms

- A matching is ***Pareto efficient*** if there is no matching which assigns each student a weakly better school and at least one student a strictly better school.

- A matching μ eliminates ***justified envy*** if there is no unmatched student–school pair *(i, s)* such that:
  - student *i* prefers school *s* to her assignment under μ and
  - student *i* has a higher priority at school *s* than some other student who is assigned a seat at school *s* under μ.

- The mechanism is ***strategy-proof*** if no student can possibly benefit by misrepresenting her preferences.

TTC

- Step 1:
  - The highest priority student of each school is assigned all slots of that school.
  - Each student points to the student (possibly himself) who is assigned (all slots of) his best choice.
  - There is at least one cycle. Corresponding trades in cycles are performed.

- In general, Step k, k>1:
  - Step 1 with remaining students and slots.

Example

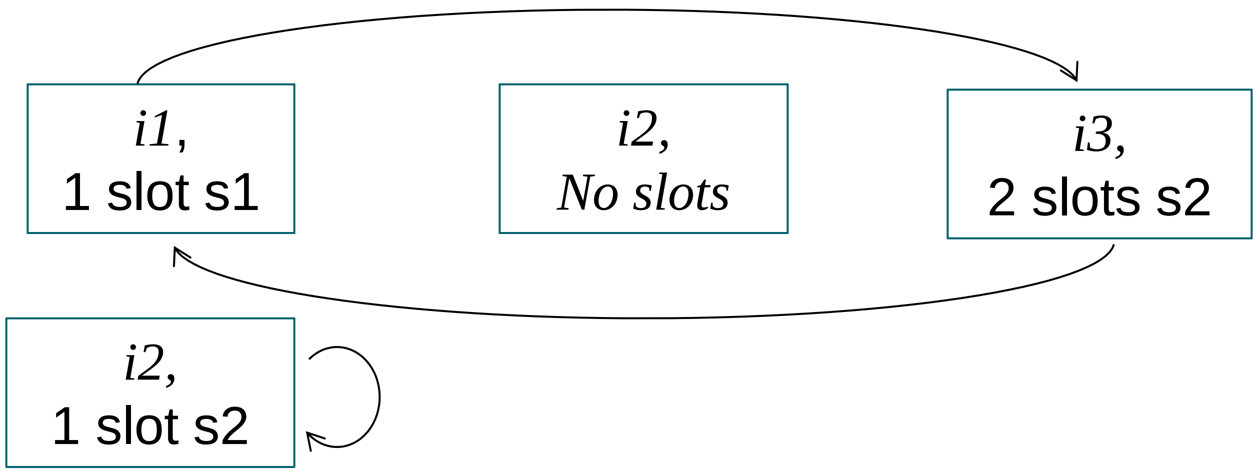
$\succ_{s_1}$	$\succ_{s_2}$
i_1	i_3
i_2	i_2
i_3	i_1

P_{i_1}	P_{i_2}	P_{i_3}
$\boxed{s_2}$	s_1	$\boxed{s_1}$
s_1	$\boxed{s_2}$	s_2

TTC:

Step 1

Step 2



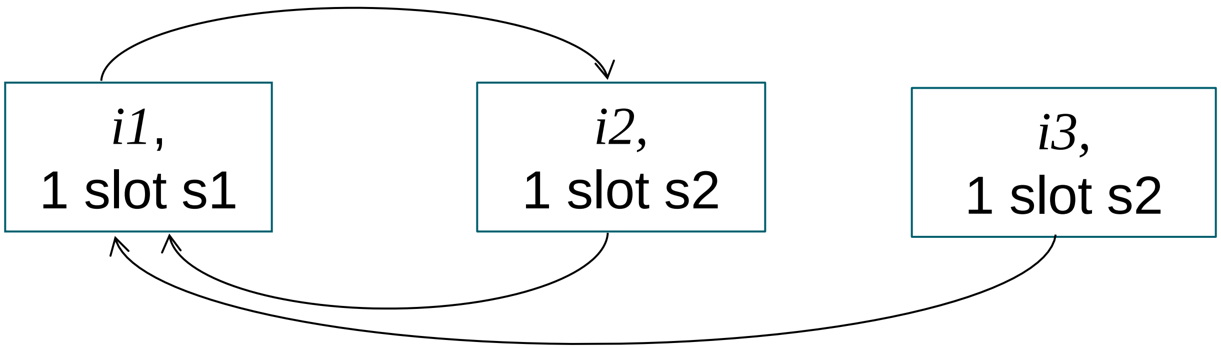
TTC result in allocation above, which is Pareto efficient, but priority of student *i2* for school s1 is violated by student *i3*.

This kind of justified envy can be avoid at no cost in terms of welfare or incentives - ETTC

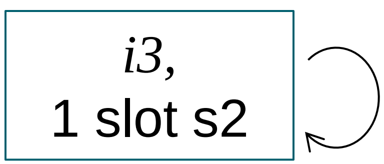
Example

ETTC:

Step 1:



Step 2:



Allocation:

S1: Student 2

S2: Studen1 2, student 3.

No justified envy

ETTC

- Step 1: For each school, all available slots are assigned to students *one by one* following their priority order to form **student-school pairs**. Each student-school pair (i; s) points to the student-school pair (i0; s0) such that:
  - (i) school s0 is the best choice of student i and,
  - (ii) student i0 is the student with the highest priority for school s among the students who are assigned a slot from school s0.

- There is at least one cycle. In each cycle, corresponding trades are performed, and all student-school pairs which participate in a cycle are removed

ETTC

- In general, Step k, k >1: For each school s such that (i) there are slots of school s which remained to be inherited from previous steps, and (ii) there are no such students who were assigned a slot of school s at a previous steps of algorithm, its slots which remained to be inherited from previous steps are assigned to the remaining students one by one following the priority order for school s to form new student-school pairs. Each student-school pair (i; s) points to the student-school pair (i0; s0) such that
  - (i) school s0 is the best choice of student i and,
  - (ii) student i0 is the student with the highest priority for school s among the students who are assigned a slot from school s0.

- There is at least one cycle. In each cycle, corresponding trades are performed, and all student-school pairs which participate in a cycle are removed.

ETTC

Proposition 1: ETTC is Pareto efficient

Proposition 2: ETTC is strategy-proof

ETTC vs TTC

Proposition 3: Suppose there are two schools. If student i is in top q_s priority group for school s. Then, under ETTC student i never has justified envy. This is not the case under TTC.

Proposition 4: Suppose there are two schools. If ETTC selects an unfair allocation for a problem, then TTC also selects an unfair allocation for the same problem. The converse is not necessarily true.

Experiment

- The aim: compare TTC and ETTC from equity criterion in lab.

- The design is based on Chen and Sönmez, 2006.
- Two treatments: TTC and ETTC
- Three environments under complete information
  - Designed
  - Random correlated
  - Random uncorrelated
- 7 sessions for each algorithm, total 140 subjects.
- Each session took about 90 min, average payoff of €15.32.

Environments

- Designed environment
  - 3 schools (3,3,4 slots). Designed to get an allocation without justified envy under ETTC.
- Random correlated environment
  - 5 school, 2 slots each. 6 students prefer schools D or E, other 4 students prefer A or C. Fair allocation is not feasible under ETTC nor TTC.
- Random uncorrelated environment
  - 4 schools (2,2,3,3 slots). Random preferences (But district school is never the first choice)

Hypothesis

Hypothesis 1:

Participants of the experiment choose to state their true preferences for allocations under both TTC and ETTC as both mechanisms are strategy proof.

Hypothesis 2:

TTC and ETTC should not differ from the efficiency criteria, as both mechanisms are Pareto-efficient.

Hypothesis 3:

On average, the number of justified envy outcomes generated by ETTC should be lower than those of TTC.

Results

Result 1 (Truthful Preference Revelation):

In all environments, the differences in proportions of truthful preference revelation under TTC and under ETTC are not statistically significant.

	Designed environment	Random-correlated environment	Random-uncorrelated environment
TTC	57%	30%	37%
ETTC	56%	27%	40%
Test of proportions, p value	0.865	0.708	0.728

Results

Result 2 (Efficiency):

In all environments, the differences in efficiency under TTC and under ETTC are not statistically significant.

Mechanism		TTC	ETTC	p-value
Designed environment	Mean efficiency Asymptotic standard error	86.38% 0.026	82.37% 0.028	0.70
Random-correlated environment	Mean efficiency Asymptotic standard error	91.37% 0.015	89.30% 0.019	0.80
Random-uncorrelated environment	Mean efficiency Asymptotic standard error	84.51% 0.001	84.65% 0.001	0.95

Result 3 (Number of justified envy outcomes):

In the designed and random-correlated environments ETTC produces significantly less justified envy outcomes than TTC does. In random-uncorrelated environment, the difference in the number of justified envy outcomes produced by ETTC and TTC is not statistically significant.

Mechanism		TTC	ETTC	p-value
Designed environment	Mean number of justified envy outcomes	4.76	2.89	0.03
	Asymptotic standard error	0.66	0.58	
Random-correlated environment	Mean number of justified envy outcomes	9.62	8.43	0.00
	Asymptotic standard error	0.61	0.16	
Random-uncorrelated environment	Mean number of justified envy outcomes	3.80	3.42	0.25
	Asymptotic standard error	0.42	0.49	

Result 4 (Number of students with justified envy):

In the designed and random-correlated environments ETTC produces significantly less students have justified envy to other students than under TTC. In random-uncorrelated environment, the difference in the number of justified envy under ETTC and TTC is not statistically significant.

Mechanism		TTC	ETTC	p-value
Designed environment	Mean number of students with justified envy outcomes	3.85	2.25	0.00
	Asymptotic standard error	0.48	0.42	
Random-correlated environment	Mean number of students with justified envy outcomes	7.04	5.61	0.00
	Asymptotic standard error	0.29	0.36	
Random-uncorrelated environment	Mean number of students with justified envy outcomes	2.97	2.98	0.98
	Asymptotic standard error	0.29	0.45	

Analises with respect to stated preferences

- Designer‘s concern
- Long-run learning

Random environment (the least favorable for ETTC)

- Find allocation by ETTC and TTC for the same preference profiles and compare

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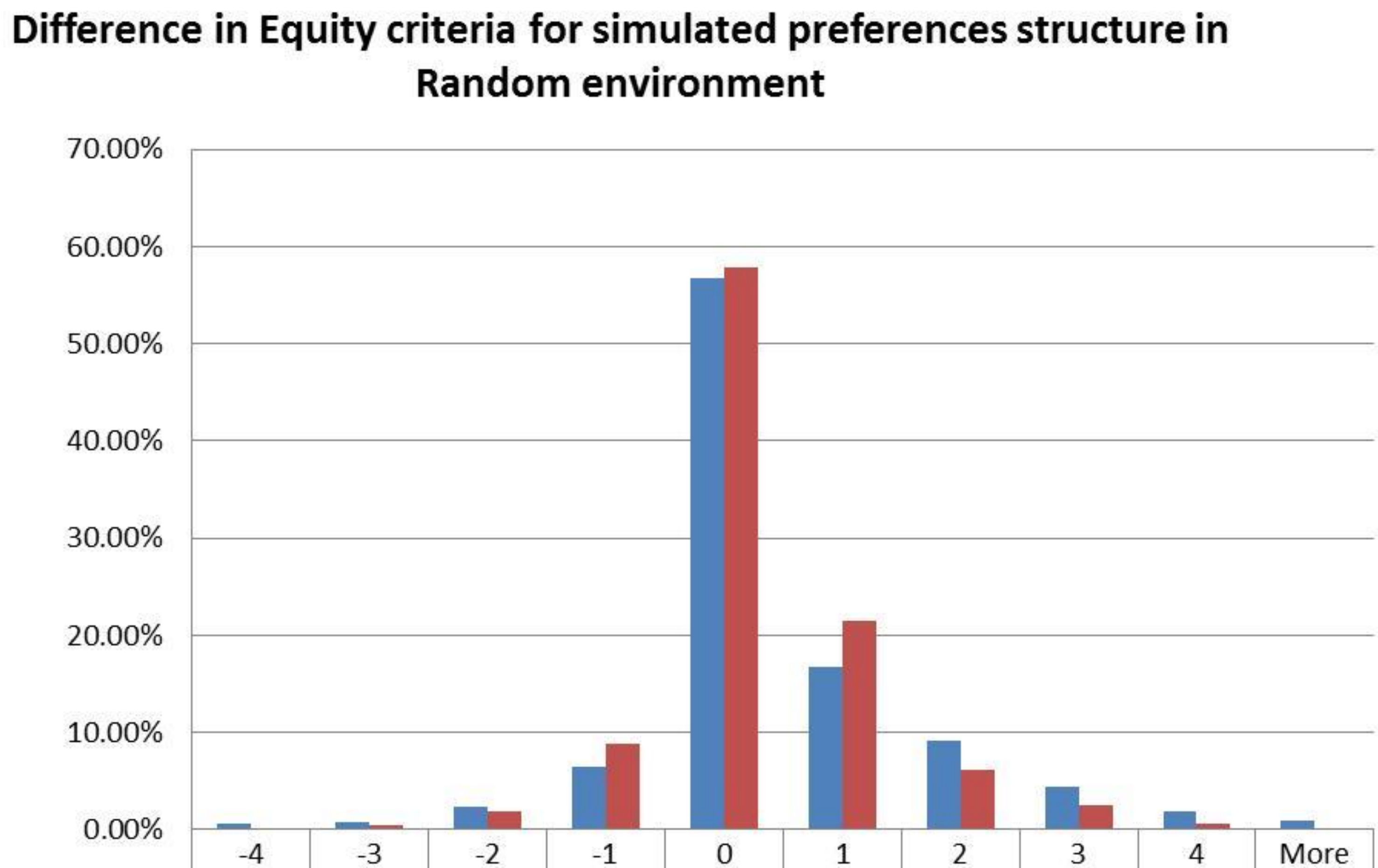
Result 5 (Equity dominance): ETTC is more likely to generate less justified envy outcomes and less students with justified envy than TTC in all environments, given the students reveal their true preferences.



	-4	-3	-2	-1	0	1	2	3	4	More
JE(TTC-ETTC)	0.65%	0.73%	2.31%	6.43%	56.74%	16.75%	9.12%	4.47%	1.83%	0.99%
UniqueJE(TTC-ETTC)	0.11%	0.48%	1.92%	8.86%	57.78%	21.42%	6.17%	2.45%	0.58%	0.22%

Conclusions

— We introduced ETTC mechanism for fairness concern.
— ETTC significantly outperforms TTC in the lab by equity criteria.
— With respect to stated preferences even in case of uncorrelated preferences ETTC on average produces significantly less number of justified envy.

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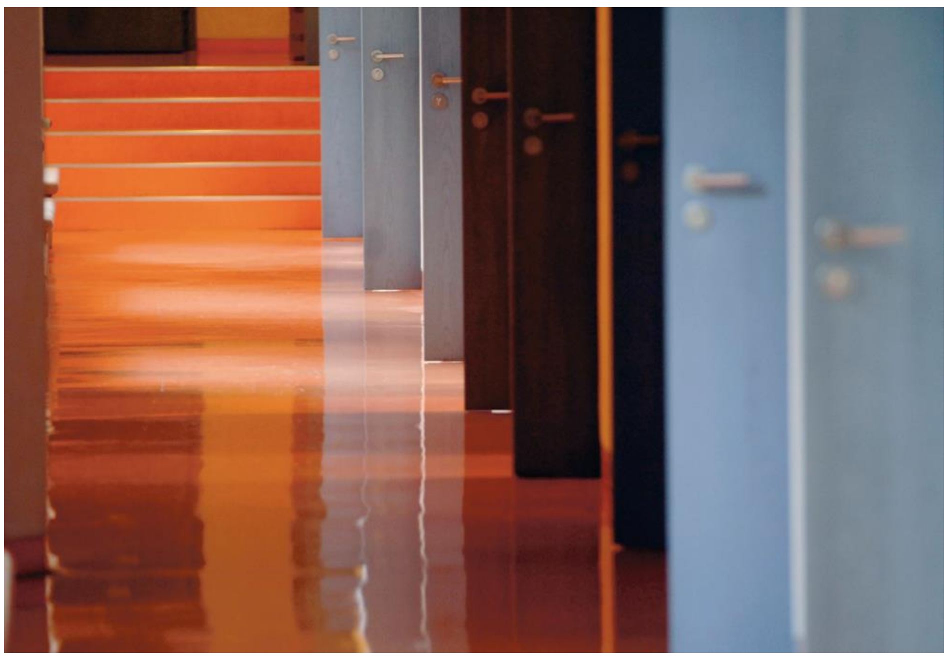
Thank you!



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