School Choice in a Tiebout Model

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- Last decades school choice has been expanded around the globe.
- Standard Tiebout models do not characterize our public finance realities.
- (and maybe they shouldn't!: Calabrese, Epple and Romano (RES2012)).

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- private schools reduce segregation in neighborhoods (starting with Nechyba (1999)).
- when free choice is introduced it is assumed that public school quality is the same in all school (Epple and Romano (2003)).

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- Families submit a ranking of schools and overdemands in a school are resolved through priority orders and specific rules (mechanisms).
- Preferences, school quality, priorities are exogenous.
- Focus on strategy proofness, stability and efficiency within this framework.

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  - Boston vs Deferred Acceptance (DA) mechanisms.
- studies the effects of private schools on the allocation of children within the public school system.

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- When no priorities for neighborhood:
  - under DA, no segregation across neighborhoods or schools,
  - under Boston, school segregation under some strong condition.

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  - Under Boston, back to segregation in schools; higher types increase their chances to access the best school.
- The specifics of the mechanism have large effects on school and neighborhood segregation and quality.
- The presence of private schools conditions the allocation *within* the public school system, affecting parents' submitted list.

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- School capacity $\eta_j = \frac{1}{3}\ \forall j$.

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- A3 captures the ghetto effect mentioned in the introduction.

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- If, again, demand $\geq$ seats left, points decide (or random draws).
- And so on until everybody is allocated a seat.

$\Rightarrow$ telling the truth is not a dominant strategy.

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$\Rightarrow$ being truthful is a dominant strategy: households submit (q_1, q_2, q_3) if $q_1 > q_2 > q_3$

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Lemma

Consider a partition T_1, T_2, T_3 of households across districts that yields $\hat{q}_1 > \hat{q}_2 > \hat{q}_3$, where $\eta_j = |T_j| \;\; \forall j$. Then both with BM and DA ranking the local school first is an undominated strategy for every household.

Residential priorities

Theorem

Under A1-A2, and both with BM and DA, there exists a unique equilibrium with $\hat{q}_1 > \hat{q}_2 > \hat{q}_3$; $T_1 = (b, 1]$; $T_2 = [a, b]$ *and* $T_3 = [0, a)$, *where where* $\eta_j = |T_j|$. *Equilibrium rents are* $r_3 = 0$; $r_2 = h(q_2, a) - h(q_3, a)$ *and* $r_1 = r_2 + h(q_1, b) - h(q_2, b)$.

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  - The ghetto emerges endogenously.
- The theorem reveals that school choice mechanisms have no effect when schools have residential priorities.

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Theorem

With DA no equilibrium with school quality differentials $q_1 > q_2 > q_3$ *or segregation exists.*

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- If agents believe that $q_1 = q_2 = q_3$ they may all rank school 1 first, school 2 second and school 3 last so that $q_1 = q_2 = q_3$ ex post.
- Moreover, this equilibrium is sequential: one can construct a sequence of beliefs (q_1^n, q_2^n, q_3^n), $n = 1, 2... \to (q_1, q_2, q_3)$ with $q_1^n > q_2^n > q_3^n)$ such that the best response profile always consists of everyone ranking school 1 first, school 2 second and school 3 last.

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- As before, the housing market does not generate any kind of segregation across districts and differences in quality across schools do not create differences in housing rents.
- Existence of equilibrium with school segregation requires one of the schools to be ex-ante perceived as the worst by every household.
- That is, it requires the existence of a sufficiently bad exogenous *ghetto school*.

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- There are two cases to consider:
  - **Case 1:** Both schools 1 and 2 give all their slots in the first round of the assignment procedure ($m_2 \geq 1/3$).
  - **Case 2:** School 1 gives all its slots in the first round of the assignment procedure while school 2 does not ($m_2 < 1/3$).

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$$V(s) = \frac{1}{3m_s} h\left(q_s, t\right) + \left(1 - \frac{1}{3m_s}\right)\left(h\left(q_3, t\right) - \Delta\right) \quad (1)$$

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- They will play strategy 1 if $V(1) > V(2)$, which can be written as:

$$\frac{h(q_1, t) - h(q_3, t) + \Delta}{h(q_2, t) - h(q_3, t) + \Delta)} > \frac{m_1}{m_2} \quad (2)$$

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  - The lemma implies the following single-crossing condition:
    - If a t-type parent chooses strategy 1 and $t' > t$, a t'-type parent also chooses strategy 1.
    - Likewise if a t-type parent chooses strategy 2 and $t' < t$, a t'-type parent also chooses strategy 2.

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- Hence, $m_1 = 1 - \Phi(\tilde{t})$ and $m_2 = \Phi(\tilde{t})$ and we can use (2) to write that threshold as:

$$\frac{h\left(q_1, \tilde{t}\right) - h\left(q_3, \tilde{t}\right) + \Delta}{h\left(q_2, \tilde{t}\right) - h\left(q_3, \tilde{t}\right) + \Delta} = \frac{1 - \Phi(\tilde{t})}{\Phi(\tilde{t})} \tag{3}$$

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- Parents playing strategy 1 have their children assigned to school 1 with probability $1/3m_1$, to school 2 with probability $\frac{m_1-2/3}{m_1}$ and to school 3 with probability $1/3m_1$.
- Their expected utility is:

$$V(1) = \frac{1}{3m_1} h\left(q_1, t\right) + \frac{m_1 - 2/3}{m_1} h\left(q_2, t\right) + \frac{1}{3m_1}\left(h\left(q_3, t\right) - \Delta\right) \tag{4}$$

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- And they will play strategy 1 if $V(1) > V(2)$, or:

$$\frac{h(q_1, t) + h(q_3, t) - \Delta}{h(q_2, t)} > 2 \tag{5}$$

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  Under assumptions A1-A3, the LHS of (4) is increasing in t.

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  - The threshold is now given by:

$$\frac{h\left(q_1, \tilde{t}\right) + h\left(q_3, \tilde{t}\right) - \Delta}{h\left(q_2, \tilde{t}\right)} = 2 \tag{6}$$

BM without priorities

Theorem

Under assumptions A1-A3, there is an equilibrium in the Boston Mechanism with no priorities nor private schools with a strategy profile characterized by a threshold $\hat{t} \in (\underline{t}, t')$ *such that all types above the threshold rank school 1 first and all types below rank school 2 first. School 3 is ranked last by every type. If* $\Phi(\hat{t}) \geq 1/3$ *this equilibrium brings full segregation between schools 1 and 2. Segregation is partial if* $\Phi(\hat{t}) < 1/3$*. Moreover, this equilibrium is sequential. If* Δ *is high enough, this sequential equilibrium is unique and entails full segregation.*

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Lemma

If $A1-2$ hold, if a household t prefers s_p to s_2 or s_p to s_3, then so does household with $t' > t$ (single-crossing wrt to private school).

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For all $t > t_2$, applying only to s_1 is the (weakly) dominant strategy.

Lemma

For all $t < t_3$, applying to s_1 provides lower expected value than in the model without private schools. That is, if $t = \tilde{t} < t_3$, applying for s_2 will be a best response.

No priorities and private schools: BM

Theorem

For any $\Delta > \Delta^*$, there are two prices of the private school, p^* and $p^{**} > p^*$, such that for all $p \in [p^*, p^{**}]$ there exists an equilibrium in BM without priorities with segregation across schools. The equilibrium is characterized by a threshold type $\tilde{t}^{priv}$ such that households with $t \geq \tilde{t}^{priv}$ play strategy 1 and those with $t < \tilde{t}^{priv}$ play strategy 2. In equilibrium $q_1^{priv} > q_2^{priv}$. Moreover, if $t_3 > \tilde{t}$ then $\tilde{t}^{priv} > \tilde{t}$.

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Corollary

In the equilibrium with private schools, if $t_3 > \tilde{t}$ *then* $q_1^{priv} > q_1$*,* $q_2^{priv} > q_2$*, and rich types have higher probability of accessing the best school.*

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- Hence, $q_1^{priv} > q_2^{priv} > q_3^{priv}$;
- if $t_2 > 1$, then $q_1 = q_2 > q_3$.
- The probability of entering each school, in equilibrium, is independent of t.

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- But if done using different lotteries or assigning open seats first, then
- we find that perfect *residential* segregation, partial *school* segregation and a quality hierarchy emerge when schools reserve a positive proportion of seats to local residents.

Conclusions

- If priorities for residence, both mechanisms lead to Tiebout equilibria with segregation in neighborhood and schools.
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- Partial priorities can lead to Tiebout in neighborhoods, partial segregation in schools.
- Boston is more vulnerable to the details of the choice problem and can easily lead to segregation.