Incentives in Landing Slot Problems*

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Abstract

We analyze three applied incentive compatibility conditions within a class of queueing problems motivated by the reassignment of flights to airport landing slots. A preexisting landing schedule becomes wasteful when airlines privately learn updates about their flights' cancelations or feasible flight times. The FAA's objective is to create a new queue that does not waste landing slots. We separately consider the airlines' incentives to report their flights' (IC1) feasible arrival times, (IC2) delay costs, or (IC3) cancelations. Our first three results show that any Pareto efficient rescheduling rule must be manipulable by *each* of these three methods separately.

By weakening efficiency to a form currently achieved by the FAA, we recover incentive compatibility with respect to (IC2) and (IC3) by extending the Deferred Acceptance (DA) algorithm, while the FAA's current mechanism fails (IC3). Our extension is consistent with the FAA's information infrastructure, which does not elicit delay cost information. We show that essentially any such rule must fail (IC1), but that our extension satisfies a weak version of (IC1).

Our model can be viewed as a one-sided version of the Gale–Shapley College Admissions model where only college preferences are relevant. This makes it a counterpart to the School Choice model (Abdulkadiroğlu and Sönmez (2003)) in which only student preferences are relevant.

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1 Introduction

While weather-caused flight delays are frustrating to individual air passengers, they are of even greater concern to policy makers. The economic impact of such delays measures in the billions of dollars per year.[1] While delays from weather are unavoidable, it turns out that some of the resulting costs can be recovered by optimally rescheduling flights that remain active during the delay after others have been canceled. This rescheduling is done through a centralized authority—the Federal Aviation Administration (FAA)—but only *after* airlines report certain privately-known information about their flights. This yields a natural mechanism design problem that we formalize in this paper. Subject to design constraints appropriate for this setting, we analyze three forms of incentive-compatibility pertaining to the reports of flight delays, cancelations, and waiting costs.

Though we use the language of this application to describe *landing slot problems*, our model can be interpreted more generally as *constrained queueing*. Agents (airlines) own sets of jobs (flights) already queued to be processed by a server (airport). Some of the jobs are to be canceled, while each remaining job has some earliest feasible processing time and waiting cost. The remaining jobs must be rescheduled to avoid the efficiency loss from gaps in the queue. For a centralized planner, doing this *feasibly* requires knowing both cancelations and feasible processing times. The agents' preferences are also determined by this—along with waiting costs—all of which can be privately known.[2]

The goal of this paper is to determine the extent to which an efficiency-improving mechanism can induce agents to truthfully report one or more types of such information. Our results are mixed, depending on the degree of efficiency one seeks to obtain. We show that a strong form of efficiency is incompatible with *any* one of our three incentive compatibility conditions. However, a weaker form of efficiency—the one considered in the transportation literature on this problem—can be achieved while satisfying two of our three IC conditions, along with a weakened version of the third. The rescheduling rule we use to demonstrate this possibility result is an adaptation of the well-known Deferred Acceptance algorithm. Though this rule is well-studied, none of our results directly follows from the related literature. In fact some of the results are more positive than in other environments.

[1] A 2008 report of the US Joint Economic Committee (Sen. C. Schumer et al.) estimated the annual economic cost of *all* flight delays to exceed \$40 billion, around half of which is direct costs to the airlines. Weather was the cause for roughly one-fifth of flight delays in January–June 2012. This is close to the historic average; e.g. see http://www.transtats.bts.gov/OT_Delay/OT_DelayCause1.asp, remembering that N.A.S. delays are primarily weather related.

[2] We consider each of these three information types separately, so any single result of ours applies regardless of whether the other two types are private information. In the landing slot application it is a reasonable approximation to assume all of them to be privately known, as we describe later.

To motivate our modeling choices and design constraints, Subsection 1.1 provides a brief background on Ground Delay Programs (GDP's)—the rescheduling process used by the FAA when inclement weather reduces landing capacity. Various institutional (and legal) constraints translate directly into mechanism design constraints pertaining not only to incentives, but forms of implicit property rights. We discuss related literature in Subsection 1.3 before formalizing our model in Section 2. A summary of our results is given in the concluding Section 7.

1.1 Ground Delay Programs

Our modeling choices, along with the design constraints we impose on mechanisms, are motivated by institutional details such as legislative requirements, the FAA's own explicit policies, and the FAA's current infrastructure for communication. We begin with basic information on how Ground Delay Programs (GDPs) are performed, referencing the details most relevant to our analysis. While we provide enough background to explain our assumptions, we refer the reader to other sources for a more extensive discussion of the history and details of the general GDP process.[3]

The purpose of a GDP is to temporarily reduce the rate of air traffic at an airport when there is not enough capacity to satisfy the projected demand for landing slots. This supply-demand imbalance is typically the result of inclement weather, during which landing rates at airports are reduced for safety reasons, effectively reducing the supply of slots. Hours in advance of a forecasted weather event for a particular airport, air traffic management declares that a GDP will go into effect for that airport. Flights that are destined for that airport—but still on the ground at their origination airports—are given delayed departure times while still on the ground. These delayed departure times are spread out in order to reduce the arrival rate at the affected airport.[4]

This description of GDP's is misleadingly simple: all that seemingly needs to be done is to spread out the arrival of flights in order to accommodate a reduced arrival rate. Indeed, this is the first step of a GDP, known as *Ration-by-Schedule*. For example, suppose the airport normally lands sixty flights per hour, but is limited to thirty flights per hour due to weather. Then thirty 2-minute slots are created where there used to be sixty 1-minute slots, and they are assigned in respective order to the first thirty flights of the originally planned landing schedule.

[3]See Wambsganns (1997), Ball, Hoffman, and Vossen (2002), or Schummer and Vohra (2012), and references therein.

[4]Flights already in the air, and certain other flights, are exempted from ground delays. This detail changes neither the essence of our description nor the validity of our results, so we ignore it.

Ration-by-Schedule *per se* would solve the supply-demand imbalance, if not for certain operating constraints of the airlines. When a flight is assigned a delayed departure time, its airline may be forced to react by canceling (or further delaying) that flight. This could be due to a crew timeout (when they exceed their legal work hours for the day), related aircraft delays at other weather-affected airports, or other reasons. Regardless of the reason, such flight cancelations—made *after* the Ration-by-Schedule procedure—have the effect of *reducing* demand for landing slots. In other words, the solution to a problem of scare resources (reduced landing slots) yields a new problem of excess resources (newly vacated slots).

Therefore, in order to make efficient use of these newly created gaps in the landing schedule, the FAA must perform the *Reassignment* step of a GDP. After soliciting relevant information from the airlines about changes to their flights' status and feasible departure (and hence arrival) times, the FAA feasibly reassigns the remaining flights in order to eliminate (or minimize) vacancies in the landing schedule. It is this step that we examine as a mechanism design problem.

Since around 1998 the FAA has used what is known as the Compression Algorithm to perform the Reassignment step of a GDP. This choice of procedure for this step is not as obvious as in the Ration-by-Schedule step for at least two reasons. First, one cannot arbitrarily move flights earlier in the schedule to fill vacant slots, since doing so might violate a flight's feasibility constraints. Second, *various institution details* yield additional constraints on the kinds of rules that can be used to reassign flights. We now elaborate on these details, since they motivate many of the definitions in Sections 3 and 5.

1.2 Motivation for Design Constraints

Our objective is to analyze various forms of incentive compatibility under the following design constraints.

Feasibility. The FAA's primary objective is to feasibly reassign flights so that no landing slots are wasted. In order to achieve this, each airline is asked to report not only its flight cancelations (if any), but also the earliest feasible arrival time for each of its remaining flights. This information allows the FAA to determine which slots are vacated and which of the remaining flights can fill them. It is sufficient information to construct a *non-wasteful* landing schedule, i.e. one that does not leave any desirable slot vacant.

Simplicity. While flight arrival information is sufficient to calculate non-wasteful landing schedules, it is not enough information to determine which landing schedules are fully Pareto efficient (see Subsection 3.1). Full airline preference information would be required to know whether an airline prefers to move flight f up to slot s, or move flight f' up to slot s'. The FAA does not currently solicit this information. Its current method for executing the Reassignment Step—the Compression Algorithm—takes as input only the cancelation and arrival time information described earlier.

Such a restriction is practical, since such preference tradeoffs would require an airline to evaluate and report preference information that is exponential in the number of flights. In Section 5 we call such rules *simple*: those that satisfy this soft design constraint of taking as input *only* cancelation and feasible arrival information.

Self-optimization. When the FAA provides a landing schedule, it specifies precisely which flight is to occupy each landing slot. Since (by *simplicity*) the airline does not report a preference for *which* of its flights should be given earlier slots, it seems obvious that an airline should be permitted to swap the positions of any of its flights within its own portion of the landing schedule. It turns out that this right is not only respected in practice, but is explicitly granted in Section 17–9–5 of the FAA's *Facility Operation and Administration Handbook*.[5] We regard this as another design constraint. Regardless of how the FAA prescribes a landing schedule, the landing schedule ultimately "consumed" by the airlines will be one in which each airline has "self-optimized" its own portion of the schedule.

All three of the above objectives are motivated by market design constraints in the FAA's problem of reassigning landing slots. In Section 5 we call a reassignment rule *FAA-conforming* if it satisfies all three of these conditions.

Individual Rationality. There is a trivial but unappealing way of satisfying the above design constraints; namely by using a serial dictatorship. Such a method first would allow airline A to occupy whichever slots it wants; subject to this it would allow airline B to occupy any of the remaining slots it can use, etc. Not only is such a method unfair, but it would obvious be disruptive each airline's internal planning as its own landing schedule continues to be arbitrarily re-specified. On the other hand, offering airlines the *option* to move its flights earlier is not disruptive, since an airline would be free to decline. There we restrict attention to rules that do not make any airline worse off than they already are following the

[5]The handbook is available through http://www.faa.gov/atpubs.

Rationing step of the GDP.[6]

1.3 Related Literature

We expand the landing slot model of Schummer and Vohra (2012) by explicitly considering airline preference for delaying one flight in exchange for expediting another. That paper and ours sit between two literatures: an operations-oriented literature on the specific issue of designing GDP protocols, and the game theoretic literature on matching.

Operations and Transportation

While the operations literature on GDP's mentions some concern for incentives, the concept is not formalized to the extent it is in the economics matching literature, as we do here.[7] This literature instead considers various (social welfare) optimization problems under the implicit assumption of complete information. For an historical perspective on this problem, see Wambsganss (1997).[8]

Vossen and Ball (2006a) provide a linear programming approach to minimize total airline costs, showing that a special case of their formulation is equivalent to Ration-by-Schedule. Compared to their approach, they also argue (with data) that the currently-used Compression Algorithm is slightly worse at compensating airlines for releasing slots. Nevertheless, given the magnitude of costs in this setting, these slight differences can add up to significant sums, justifying further analysis.

Both the previous paper and Vossen and Ball (2006b) interpret the Compression Algorithm as an implementation of a barter exchange process (under the implicit assumption of truthful behavior). Following this comparison, they suggest improvements that would increase efficiency from further trade.

Various papers extend this line of research into (social welfare) optimization in GDP's. Ball, Dahl, and Vossen (2009) enhance optimization by centrally endogenizing flight cancelation decisions. Hoffman and Ball (2007) add banking constraints, requiring certain "connected" pairs of flights to land within a given time of each other. Niznik (2001) includes a downstream effect of arrival delays, such as resulting departure delays. Ball and Lulli (2004)

[6]There are a few reasonable ways to define such a condition, but our results are robust to that choice. A stronger condition than we consider—the core—is discussed by Schummer and Vohra (2012), whose main motivation is the concept of property rights.

[7]An interesting exception is a rigorous, explicit example of manipulability provided by Wambsganss (1997). Making it all the more interesting is the fact that the article's main purpose is to provide history and motivation for the FAA's mid-1990's modification of its rescheduling rule.

[8]Many of the references in this section also provide background information.

and Ball, Hoffman, and Mukherjee (2010) modify Ration-by-Schedule by discriminating the schedule in favor of distant flights; this improves efficiency when there is uncertainty about future airport capacity.

Beyond optimization, this literature also addresses *equitable* allocation amongst airlines. Vossen and Ball (2006a) show that Ration-by-Schedule lexicographically minimizes the maximum delay among all flights relative to their pre-GDP landing times, i.e. a Lorenz domination result. Manley and Sherry (2010) examine both performance and equity measures for various other methods of rationing slots.

Matching

The other related literature is that of matching problems and object allocation. With airlines exchanging endowed landing slots, our model could be seen as a generalization of the (1-sided) housing market model of Shapley and Scarf (1974). Given the nature of our results, however, it is more informative to see our model as a restricted form of the celebrated (2-sided) College Admissions model of Gale and Shapley (1962). They propose the well-known Deferred Acceptance algorithm to match each capacity-constrained “college” (analogously, airline) to its own set of “students” (landing slots) in a way that is stable.[9]

Our main restriction to the Gale–Shapley model is that the students (landing slots) in our setting do not have preferences while colleges (airlines) do. In this sense our model fills a gap in the literature as a complement to the School Choice model of Abdulkadiroğlu and Sönmez (2003) in which students have preferences but colleges do not.[10] In these applications, colleges (or schools) have priority orders over students that can , for example, play the role of preferences in the Deferred Acceptance algorithm. The “stability” of the resulting match can be reinterpreted as a fairness condition with respect to the priority orderings. Hence, to the extent that the priorities are meaningful, such priorities-based models lie somewhere between the classic 1- and 2-sided models.

Incentives have been studied extensively in these models. Under the student-proposing version of the Deferred Acceptance algorithm (sDA) proposed by Gale and Shapley (1962), it is a weakly dominant strategy for students to report their true preferences (Dubins and Freedman (1981); Roth (1982)). On the other hand, Roth (1982) shows that if a rule is stable, then it cannot give such an incentive to all students *and* colleges.[11]

[9]See Roth and Sotomayor (1990) for a definition of stability and other standard concepts used in this introduction.

[10]See also Balinski Sönmez, Sönmez(2011), Sönmez and Switzer (2012), and Kominers and Sönmez (2012).

[11]Furthermore, unless all colleges have unlimited capacities, not even an efficient, individually rational rule provides such incentives; see Sönmez (1996), Alcalde and Barberà (1994), and Takagi and Serizawa (2010).

Restricting to the School Choice model—where implicity only student incentives matter—attention is focused on sDA (in light of the above results) and Top Trading Cycle (TTC) algorithm of Shapley and Scarf (1974) which is not only strategy-proof but Pareto-efficient. While sDA is generally not efficient; Ergin (2002) shows that it is when school priorities satisfy an acyclicity condition. On the other hand Kesten (2006) shows that a further priority domain restriction makes TTC stable, equating the two mechanisms. Even on other domains where sDA is inefficient, sDA cannot be Pareto-dominated by another strategy-proof rule.[12]

An important fact to highlight is that, despite the positive results regarding student incentives, it is more problematic to construct rules that provide incentives for the college side of the classic model, e.g. as shown by Roth (1985). This suggests a greater challenge within our model, since that is the side of the market we consider as agents, while the student-side plays the role of objects (landing slots) for us. Nevertheless, we provide a class of rules that satisfies a partial list of positive incentives properties, as we show in Subsection 5.3 and Section 6.

In our model, airlines have initial ownership over slots before they are exchanged. This feature is reminiscent of the model of house allocation with existing tenants (Abdulkadiroglu and Sonmez (1999)). This model yields an individually rational, Pareto-efficient, strategy-proof mechanism. Our model differs from that one in a few ways, most significantly in terms of our agents' specific form of preferences for multiple "houses" (landing slots).

In models where agents consume *sets* of objects (as opposed to a single house or college), results tend to be negative. Konishi et al. (2000) endow agents with indivisible objects and show that even with additive preferences, the weak core can be empty. In a related model with separable preferences over sets, Atlamaz and Klaus (2007) show that efficient, individually rational rules must be manipulable through various forms of destroying, concealing, or transferring endowed objects.[13] We define similar conditions in Section 3. Our Theorem 3 is a parallel result to one in Atlamaz and Klaus (2007), neither implying nor implied by theirs. More significantly we provide a contrasting positive result in Subsection 5.3

These impossibility results have no implication in our model due to our structure on preferences.

[12]See Abdulkadiroglu, Pathak, and Roth (2009), and also Kesten (2010).

[13]Various other forms of endowment-manipulation are considered following the seminal contribution on the subject by Postlewaite (1979). Somewhat distant from ours is the 2-sided matching model of Sertel and Ozkal-Sanver (2002), who augment the marriage model by giving each agent a private endowment of money that is shared the mate through fixed sharing functions. The mostly-negative results in their model—on the manipulability of deferred acceptance through endowment hiding or destroying—contrast particularly with our positive result in Subsection 5.3.

2 Model

We introduce a model of time slot allocation in the language of the one application discussed above: the reassignment of airport landing slots to airlines, in which each airline wishes to use those slots for its own set of flights. Despite our use of this language, it should be understood that the model is applicable to any environment in which jobs must be rescheduled for processing when other jobs have been canceled or delayed.

There is a finite set of airlines $\mathcal{A}$, with elements typically denoted $A, B, C \in \mathcal{A}$. The goods to be allocated to the airlines consist of an ordered set of slots described by a (countably infinite) set of integers $S \subset \mathbb{N} \equiv \{1, 2, \ldots\}$ with generic elements $s, s', t \in S$.[14] The slot labels have cardinal meaning, e.g. slot 1 is interpreted as the earliest slot, slot 5 is two units of time later than slot 3, etc. This plays a role in the formalization of preferences, below.

Each airline $A \in \mathcal{A}$ is associated with its own finite set of flights F_A, with the set of all airlines' flights denoted $F = \bigcup_{A \in \mathcal{A}} F_A$.Though we speak of allocating slots to airlines, this is done with the ultimate purpose of matching flights to slots. Each flight requires the use of a single slot which cannot be shared. In addition, each flight $f \in F$ must be assigned to a slot no earlier than its **earliest arrival time** e_f.

Flights are assigned to slots in a **(flight) landing schedule**, which is a function $\Pi \colon F \to S$ that is injective ($f \neq f'$ implies $\Pi(f) \neq \Pi(f')$). A landing schedule is (time) **feasible** if for all $f \in F$, $\Pi(f) \geq e_f$.

A landing schedule implicitly describes which airlines take ownership of the slots occupied by flights, but does not specify any form of ownership or endowment rights over vacant (unoccupied) slots. In order to completely specify which airlines possess which slots, we require the concept of a **slot ownership function**, which is correspondence $\Phi \colon \mathcal{A} \to S$ that partitions S by airline; i.e. such that $A \neq B$ implies $\Phi(A) \cap \Phi(B) = \emptyset$. Since $\Phi(s)$ is the airline that owns slot s whether it is occupied or not, we say that Φ is consistent with a landing schedule Π when

$$\forall A \in \mathcal{A}, \forall f \in F_A \quad \Pi(f) \in \Phi(A).$$

Any (Π, Φ) satisfying this consistency condition is called an **assignment**.

[14]For all practical purposes, the set of slots is finite. We assume it to be infinite only to make the problem well-defined when airlines potentially misreport flights' feasible arrival times.

Preferences

The strategic agents of our model—the airlines—consume subsets of S. Rather than having general, arbitrary preferences over such sets, it is natural to model the following two preference restrictions.

First, airlines care only about landing their flights. Therefore an airline A has preferences only over subsets of size $|F_A|$. Strictly smaller subsets would violate feasibility. Preferences over sets of size $|F_A|$ can be implicitly extended to larger sets on the basis of which $|F_A|$ slots within the set the airline would actually use. Observe that it therefore makes sense to say that airlines have preferences over landing schedules.

A second restriction concerns the time element of the model. We assume that, all else being equal, an airline A would prefer to have flight $f \in F_A$ land as early as possible, subject to being on earlier than its earliest arrival time e_f. Assigning f to a slot earlier than e_f violates feasibility. However airline A considers it strictly better to have f assigned to slot $s \geq e_f$ than to have f assigned to slot $t > s$ (holding other flight assignments fixed).

This second restriction says nothing about how an airline evaluates the tradeoff between moving flight $f \in F_A$ to an earlier (feasible) slot in exchange for moving $g \in F_A$ to a later slot. Indeed one can imagine various classes of preferences that would express such a tradeoff.[15] As a starting point, we consider a preference model reflecting the idea that each flight has its own delay cost in landing which is linear in delay, and that airlines aggregate these costs in order to evaluate landing schedules.

Specifically, each flight $f \in F_A$ has a **weight** $w_f > 0$ that reflects the cost incurred by A for each unit of time that flight f remains in the air. In fact it is more precise to think of w_f as a *relative* cost. Since there is no money in our model, this "cost reduction" is used only to evaluate the tradeoff a single airline would face in moving f to an earlier slot in exchange for delaying some $g \in F_A$ to a later slot.[16] As an example, suppose A, with $F_A = \{f, g\}$, wishes to evaluate two landing schedules, Π and Π' where $\Pi(f) = 5$, $\Pi(g) = 6$, $\Pi'(f) = 3$, $\Pi'(g) = 9$. In moving from Π to Π', A is affected in two ways: a benefit of $2w_f$ units for moving f up two slots, but a cost of $3w_g$ units for moving g down three slots. Therefore

[15] One can also imagine many real world details that would complicate such evaluations, such as whether a flight has to meet some secondary deadline in order for passengers to make various connections, deadlines after which pilots must take forced rest periods, etc. While such considerations probably exist, it seems that accommodating such minor details in a real world setting would add considerable cost in terms of complexity and practicality. Therefore we omit them.

[16] While weights yield preferences for each individual airline, we make no assumption about the comparability of weights *across* airlines. One could interpret weights to represent actual dollar costs of delay, in which case one may wish to analyze social welfare by minimizing total weighted delay across *all* airlines. This is the focus of much of the operations literature discussed in Subsection 1.3. We focus instead on incentives and property rights, so we need not assume that these weights are comparable across airlines.

airline A strictly prefers Π' to Π when $2w_f - 3w_g > 0$.

Definition 1. A list of weights $(w_f)_{f \in F}$ induces for each airline $A \in \mathcal{A}$ a **preference relation**, $\succeq^w_A$, over feasible landing schedules as follows.

$$\text{For all feasible landing schedules } \Pi, \Pi': \qquad \Pi \succeq^w_A \Pi' \quad \Leftrightarrow \quad \sum_{f \in F_A} w_f(\Pi'(f) - \Pi(f)) \geq 0.$$

Furthermore, if Π is feasible and Π' is infeasible for A, then $\Pi \succ^w_A \Pi'$.

We only consider the implementation feasible landing schedules, and it is easy to see that Definition 1 uniquely defines a complete, transitive relation over such schedules. On the other hand our analysis of *incentives* creates the possibility that, by *misreporting* certain information, an airline's flight could be assigned to an infeasible slot. To handle this, it is enough for us to assume that any infeasible landing schedule is strictly worse than any feasible landing schedule.[17] Finally note that such preferences are selfish, in that an airline A does not care how other airlines' slots are allocated amongst themselves.

While there could be more general ways to model airline preferences,[18] we believe that any reasonable model analyzing delay tradeoffs among flights should at least *contain* our "linear-weight" preferences as a special case, i.e. linear preferences cannot be ruled out ex ante. Fortunately our results are robust under this position. First, following standard arguments in mechanism design with restricted preference domains, *our negative results are considerably strengthened* by restricting our domain to linear-weight preferences. Any negative result on our domain can be immediately extended to any superdomain. Second, we show that the rule yielding our possibility results continues to work on significantly larger preference domains. We discuss this in the concluding Section 7.

3 Reassignment Rules and their Properties

To summarize the primitives so far, an **Instance (of a Landing Slot Problem)** is a tuple

$$I = (S, \mathcal{A}, (F_A)_{A \in \mathcal{A}}, e, w, \Pi^0, \Phi^0)$$

of slots, airlines, flights, earliest arrival times, flight weights, and an *initial* assignment (Π^0, Φ^0). The initial assignment is relevant only to the extent that one wishes to respect

[17] This does not uniquely determine relative preferences over infeasible schedules, but this turns out not to be relevant in the analysis.

[18] E.g. In the analysis of a landing schedule optimization problem, Ball and Lulli () consider convex delay costs.

some form of property rights with respect to such an endowment.[19]

A **reassignment rule** is a function φ that maps each instance $I = (S, \mathcal{A}, (F_A)_{A\in\mathcal{A}}, e, w, \Pi^0, \Phi^0)$ into a landing schedule $\varphi(I)$ that is feasible for I. We write $\varphi_f(I)$ to be the slot to which flight f is assigned under instance I. (We may also write $\Pi = \varphi(I)$ in which case $\Pi(f) = \varphi_f(I)$.)

Our objective is to find reassignment rules that improve efficiency while respecting property rights and incentivizing airlines to truthfully report their respective parameters of the instance.

3.1 Efficiency

The primary objective of the FAA's Reassignment step in GDP's is to avoid wasting slots that have been vacated by canceled flights. Moving a flight into an earlier, (feasible) vacated slot unambiguously improves efficiency. The following definition is a straightforward formalization of this.

Definition 2. A reassignment rule φ is **non-wasteful** if, for any instance I, there exists no flight $f \in F$ and no slot $s \in S$ such that both $e_f \leq s < \varphi_f(I)$ (f can feasibly move up) and $\varphi^{-1}(s) = \emptyset$ (s is vacant).

While nonwastefulness is a reasonable starting point in terms of efficiency, we begin our analysis in Section 4 with the following standard notion, which is stronger.

Definition 3. A reassignment rule φ is (strongly) **Pareto-efficient** if for each instance $I = (S, \mathcal{A}, (F_A)_{A\in\mathcal{A}}, e, w, \Pi^0, \Phi^0)$ there exists no landing schedule Π' such that (i) for all $A \in \mathcal{A}$, $\Pi' \succeq_A^w \varphi(I)$, and (ii) for some $A \in \mathcal{A}$, $\Pi' \succ_A^w \varphi(I)$.

One could consider an even stronger notion of economic efficiency: If weights represent the actual monetary cost of flight delays per unit of time, then social efficiency would minimize the sum of costs across all airlines. It is easy to see that such a requirement would be essentially incompatible with any form of incentive condition in which airlines are free to report these weights. Furthermore, without the use of monetary transfers this concept of efficiency would not respect any minimal level of individual rationality or property rights. Therefore we do not consider such a stronger condition.

[19]Furthermore, all of our results hold whether or not one assumes that the initial landing schedule is feasible.

3.2 Property rights

A standard property rights requirement in mechanism design—commonly called *individual rationality*—guarantees an agent an outcome at least as good as its initial endowment. There are two immediate ways to interpret this requirement in the context of landing slot problems, depending on whether one allows an airline to first "optimize" the way it uses the slots it initially owns.

Consider an initial assignment (Π^0, Φ^0) of some instance I. Airline A's flights are initially assigned via the landing schedule Π^0, so if a rule outputs $\varphi(I)$, one should minimally require $\varphi(I) \succeq_A^w \Pi^0$. This requirement is weak when one realizes that the initial schedule Π^0 may not optimally schedule A's flights within A's own slots, or that A may own additional (vacant) slots under Φ^0 to which A could schedule a flight. It therefore would be quite reasonable to require that a rule weakly improve an airline's welfare even after the airline optimizes the initial scheduling of its own flights *within* its own portion of the landing schedule.[20]

We formalize only the former (weaker) version of the condition for two reasons. First, it technically strengthens our negative results. Second, it can easily be checked that our possibility results would continue to hold under the stronger version of the condition, as we discuss in Remark 2 following Theorem 5.

Definition 4. A reassignment rule φ is **individually rational** if, for any instance $I = (S, \mathcal{A}, (F_A)_{A\in\mathcal{A}}, e, w, \Pi^0, \Phi^0)$ and any airline A, we have $\varphi(I) \succeq_A^w \Pi^0$.

3.3 Incentives

We consider various incentive properties that may be imposed on reassignment rules. Most of these properties compare the output of a rule following some change in the parameters of an instance I, e.g. the effect of an airline changing the report of its weights within list w, or arrival times in e, etc. Therefore it is helpful to introduce the following "replacement notation." For any instance $I = (S, \mathcal{A}, (F_A)_{A\in\mathcal{A}}, e, w, \Pi^0, \Phi^0)$, we write

$$I_{w\to w'} \equiv (S, \mathcal{A}, (F_A)_{A\in\mathcal{A}}, e, w', \Pi^0, \Phi^0)$$

to be the instance which is simply I, but with weights w replaced with w'. Similar meanings apply to $I_{e\to e'}$, etc.

The preferences of airlines are impacted directly by their flights' weights w, but also by their earliest arrival times e. We separately consider the consequences of an airline

[20] As discussed in Subsection 1.1 and Footnote 5, this right to "optimize" is explicitly granted by the FAA.

misreporting these two types of information.

Definition 5. A reallocation rule φ is **manipulable via earliest arrival times** (or e-**manipulable**) if there is an instance $I = (S, \mathcal{A}, (F_A)_{A\in\mathcal{A}}, e, w, \Pi^0, \Phi^0)$, airline $A \in \mathcal{A}$, flight $f \in F_A$, and an earliest arrival time e'_f such that

$$\varphi(I_{e\to e'}) \succ^w_A \varphi(I)$$

where e' is obtained from e by replacing e_f with e'_f.

Definition 6. A reallocation rule φ is **manipulable via weights** (or w-**manipulable**) if there is an instance $I = (S, \mathcal{A}, (F_A)_{A\in\mathcal{A}}, e, w, \Pi^0, \Phi^0)$, airline $A \in \mathcal{A}$, flight $f \in F_A$, and weight w'_f such that

$$\varphi(I_{w\to w'}) \succ^w_A \varphi(I)$$

where w' is the same as w except with w_f replaced by w'_f.

The applicability of the above two conditions depends on the degree to which these parameters are observable to the planner. In problems where both e and w are privately known only by their respective airlines, both incentives conditions are important. Interpretation can be important here. For example, if one interprets weights w as merely the (observable) fuel cost of keeping a particular type of aircraft in the air, then w-manipulability may be viewed as less important. But one might also consider other privately known components to enter w, such as the potential need to change flight crews, future use of the aircraft, etc. If these factors are privately known, the condition becomes more important.

Our last main incentives property concerns the "report" of the objects themselves being reallocated: slots. Slots are vacated when an airline decides to cancel a flight. When this cancelation is sufficiently timely (i.e. before a reallocation rule is applied), the slot is recognized as vacant, and it can be reallocated by the rule. On the other hand, if the flight cancelation occurs sufficiently late, there can be the possibility that the slot goes unused (e.g. if no other flights depart from their airports sufficiently early so as to be able to arrive in time to use the vacated slot).

If an airline possess a slot that it knows it cannot use, it should obviously be given the incentive to reveal that information so that the slot can be used by another airline. It would then be perverse if an airline were to find that it could benefit from *failing* to reveal that a particular slot is going to go unused.[21]

[21] Interestingly this concern was raised in an internal Department of Transportation memo cited in Schummer and Vohra (2012).

To rule out this sort of potential for manipulation, we consider two separate conditions. The first appears here, while the second is given in Subsection 5.3. In words, this condition states that a manipulation occurs whenever an airline can improve its landing schedule by permanently destroying a vacant slot that it initially owns.

Definition 7. A reallocation rule φ is **manipulable via slot destruction** if there is an instance $I = (S, \mathcal{A}, (F_A)_{A\in\mathcal{A}}, e, w, \Pi^0, \Phi^0)$, airline $A \in \mathcal{A}$, and slot $s \in S$ such that

(i) $s \in \Phi^0(A)$ (A owns s),

(ii) $\nexists f \in F$ such that $\Pi^0(f) = s$ (s is initially vacant), and

(iii) $\varphi(I_{S\to S\setminus\{s\}}) \succ^w_A \varphi(I)$.[22]

We should clarify the connection between slot destruction and an airline's failure to report a flight cancelation. Imagine that an airline claims a "dummy flight" d (which it intends to cancel) occupies slot s, and that $e_d = s$ with arbitrarily large w_d. Any *individually rational* rule would have to continue to assign d to s while (possibly) reassigning other flights. Since this effectively removes d and s from the economy, the condition removes the potential for any such potential manipulation.

Thinking about a more dynamic model—in which reallocation rules could be applied iteratively—leads to a natural question: If an airline announces the cancelation of flight d only *after* the reallocation rule is applied, then what ultimately happens to slot s? At the time d is canceled, the owner of s can keep s for another of its flights, if feasible, so it is never permanently destroyed. This argument would imply that Definition 7 (iii) is too strong in that it does not allow A to move one of its flights from its slot in $\varphi(I_{S\to S\setminus\{s\}})$ to s.

Using concepts introduced in Subsection 5.3 we allow for this, leading to a weaker definition of manipulation (i.e. a stronger incentive compatibility condition) in Definition 13. Because it leads to possibility, that result also is stronger in the use of Definition 13.

4 Efficient Rules and Manipulability

We begin by considering the tradeoff between efficiency and various forms of incentive compatibility. As we see below, the tradeoff is stark. We independently consider three separate

[22]To be clear, $I_{S\to S\setminus\{s\}}$ is instance I but without slot s, so s is also deleted from Φ^0).

forms of manipulation, and show that any one of them is incompatible with Pareto-efficiency, as long as we respect some minimal level of property rights.[23]

While there are plenty of models in which efficiency conflicts with axioms such as strategy-proofness, it is worth pointing out how the results of this section are particularly strong. First, our model specifies a fairly restrictive set of preferences. While airlines have preferences over sets of slots, we assume that they are representable by a linear weighting. In addition there is some commonality of preferences among airlines, in that "earlier slots are better" (subject to time feasibility). Relative to many models in the literature, this yields a relatively narrow domain of preferences.

Secondly and perhaps more importantly, we are restricting airlines' presumed flexibility in manipulating, by considering the consequences of only a *single* manipulability condition at a time. To elaborate on this point, an airline's preference over sets of slots depends both on earliest arrival times (e) and flight weights (w). The analog of a full *strategy-proofness* condition in this model would allow airlines the flexibility to misreport either (or both) of these parameters, since both of them are components of a preference relation. When we allow an airline to misreport only one of these variables we restrict the dimension in which an airline misreports its preferences. In this sense, the results of this section are stronger than analogous results in the literature.[24]

Our first result shows that, under the basic requirement of *individual rationality*, *Pareto-efficiency* leads to e-manipulability.

Theorem 1. *If a rule is* Pareto-efficient *and* individually rational*, then it is* manipulable via earliest arrival times.

Proof. Consider the instance I described as follows.

Slot	Flight	Airline	Earliest	Weight
1	b_1	B	1	1
2	a_2	A	1	3
3	a_3	A	1	2
4	b_4	B	3	1.5
5	a_5	A	5	1.75
6	b_6	B	5	1.75

[23] The uninteresting examples of (sequentially) dictatorial rules, which ignore initial landing schedules, can be cited to achieve both efficiency and non-manipulability of all kinds, at the extreme cost of depriving airlines any level of property rights whatsoever.

[24] Cite impossibility of spf+stable rules.

There are four *individually rational* ways to allocate three slots to airline B that are feasible with respect to the e_f's. Assigning either $\{1,4,6\}$ or $\{2,3,6\}$ to B would be Pareto-dominated by assigning $\{2,4,5\}$ to B. Assigning either $\{2,4,5\}$ or $\{3,4,5\}$ to B would yield an efficient, individually rational assignment.

Let φ be an *individually rational*, *Pareto-efficient* rule. We must have $\varphi_B(I) \in \{\{2,4,5\},\{3,4,5\}\}$.

Case 1: $\varphi_B(I) = \{2,4,5\}$. In this case a_2 is assigned to slot 1, a_3 is assigned to slot 3, and a_5 is assigned to slot 6.

Let I' denote the instance that is identical to I except that A reports $e'_{a_2} = 2$ (without altering the other earliest arrival times). There is only one *Pareto-efficient* and *individually rational* assignment for I': a_3 is assigned to slot 1, a_2 is assigned to slot 2, and a_5 is assigned to slot 6. Airline A prefers this assignment to the one he receives under $\varphi(I)$. Therefore A is able to gain from misreporting e_{a_2}.

Case 2: $\varphi_B(I) = \{3,4,5\}$. In this case b_4 is assigned to slot 3, b_1 is assigned to slot 4, and b_6 is assigned to slot 5.

Let I' denote the instance that is identical to I except that B reports $e'_{b_6} = 6$ (without altering the other earliest arrival times). There is only one *efficient* and *individually rational* assignment for I': b_1 is assigned to slot 2, b_4 is assigned to slot 3, and b_6 is assigned to slot 6. Airline B prefers this assignment to the one he receives under $\varphi(I)$.

In both cases φ is manipulable by misreporting earliest arrival times. □

Remark 1. It is worth noting an implicit assumption in the statement and proof of Theorem 1 which gives it a stronger interpretation. Under our definitions, when an airline misreports its earliest arrival times, it is required to abide by whatever landing schedule is output by the rule. One can imagine a stronger non-manipulability condition which would discourage an airline from misreporting information when it can subsequently rearrange flights amongst the slots it has been allocated. Such a definition may be more appropriate since airlines are permitted to perform such rearrangements (see also Definition 8) in the real world. Allowing for this type of 2-step manipulation would result in an even stronger non-manipulability condition. On the other hand, such manipulation may be more easily detectable (since manipulating airlines would more frequently be reshuffling their schedules in ways that would initially appear to be infeasible or inefficient). Regardless of this possibility, Theorem 1 shows that Pareto-efficient rules are manipulable *even if airlines cannot reshuffle their flights* after the mechanism has operated. A similar observation applies to the other two theorems in this section.

The second way for an airline to misreport preference-relevant information is through weights w. We show that such manipulations also are a consequence of *Pareto-efficiency* and *individual rationality*.

Theorem 2. *If a reassignment rule is* Pareto-efficient *and* individually rational*, then it is* manipulable via weights*.*

Proof. Consider the instance I described as follows.

Slot	Flight	Airline	Earliest	Weight
1	b_1	B	1	1
2	a_2	A	1	4
3	a_3	A	2	4
4	a_4	A	4	3
5	b_5	B	4	3

Let φ be an *individually rational*, *efficient* rule. It is straightforward to verify that there are only two *efficient* and *individually rational* assignments for I. In one, B's flights are assigned slots 2 and 4; in the other, they are assigned slots 3 and 4. Clearly B prefers the former and A prefers the latter. We show that regardless of which is selected by φ, one of the airlines can manipulate φ.

Case 1: $\boldsymbol{\varphi_B(I) = \{3,4\}}$. Let I' denote the instance that is identical to I except that B reports a weight of $w_1' = 2$ (without altering the other weights). There is only one *efficient* and *individually rational* assignment for I': B is assigned slots 2 and 4. Since $\varphi_B(I') = \{2,4\}\ P_B\ \{3,4\} = \varphi_B(I)$, airline B benefits from misrepresenting w_1 at I.

Case 2: $\boldsymbol{\varphi_B(I) = \{2,4\}}$. Let I' denote the instance that is identical to I except that A reports a weight of $w_2' = 2$ for flight (without altering the other weights). There is only one *efficient* and *individually rational* assignment for I': B is assigned slots 3 and 4. Since $\varphi_A(I') = \{1,2,5\}\ P_A\ \{1,3,5\} = \varphi_A(I)$, airline A benefits from misrepresenting w_2 at I.

In either case φ is manipulable via misreported weights. □

Our third result in this section concerns an airline's potential incentive to effectively remove a slot from the pool of slots being re-assigned. If an airline has the ability to remove slots from the system, we show that any *Pareto-efficient* and *individually rational* rule must, at some instance, give an incentive to do this. Our proof uses an example that hinges on a potential 3-airline trade of six slots. Two airlines (A and B) would gain by this trade, but the third airline (C) would lose. However, both A and B own vacant slots, either or both of which can be used to compensate C for his loss in the 3-way trade. An *efficient* rule thus forces either A or B (or both) to "pay" C to partake in the trade. But by destroying its slot, A (or B) can make C's compensation too high to pay (with respect to *individual rationality*, forcing the *efficient* rule to make only the other airline (B or A) to compensate C instead.

Theorem 3. *If a reassignment rule is* Pareto-efficient *and* individually rational*, then it is* manipulable via slot destruction*.*

Proof. Consider the instance I described in Table 1. Note that airline D plays the role of a "dummy airline," in that D's flights already occupy their most preferred slots. *Individual rationality* thus forces a rule not to move any of D's flights.

Slot	Flight	Airline	Earliest	Weight
1	a_1	A	1	$w_1 = 3$
2	b_2	B	1	$w_2 = 4$
3	b_3	B	3	$w_3 = 3$
4	c_4	C	3	$w_4 = 2$
5	c_5	C	5	$w_5 = 3$
6	a_6	A	5	$w_6 = 4$
7	a_7	A	7	$w_7 = 0.3$
8	d_8	D	8	$w_8 = 1$
9	d_9	D	9	$w_9 = 1$
10	$\emptyset_a$	A		
11	c_{11}	C	7	$w_{11} = 0.35$
12	b_{12}	B	12	$w_{12} = 0.3$
13	d_{13}	D	13	$w_{13} = 1$
14	d_{14}	D	14	$w_{14} = 1$
15	$\emptyset_b$	B		
16	c_{16}	C	16	$w_{14} = 0.35$

Table 1: Main example in the proof of Theorem 3.

Within slots 1–6, three pairwise trades amongst airlines A, B, and C are possible. If all three are performed, A and B each gain 1 unit while C loses 1 unit. There are two ways to compensate C for this loss. One is for A to move a_7 down to slot 10, giving c_{11} slot 7. (What plays a role later, however, is that A would not be willing to move down to slot 11.) Similarly B could compensate C via slot 12. We shall show that if A is compensating C at slot 7, then A can gain by destroying slot 10. A similar argument applies to B and slot 15.

Let φ be an *individually rational*, *Pareto-efficient* rule. It can be checked that only the three landing schedules in Table 2a satisfy the conditions corresponding to *individual rationality* and *Pareto-efficiency* for instance I, so $\varphi(I)$ must be one of them. The bottom of the table shows the relative gain in terms of weights for each airline, relative to the initial landing schedule.

We show that regardless of which of the three landing schedules is selected by φ, airline A or airline B can manipulate φ by destroying its vacant slot. First suppose that $\varphi(I) \in \{\Pi^2, \Pi^3\}$. Let $I_{S \to S \setminus \{10\}}$ be the instance obtained from I by destroying A's slot 10. There

Slot	Π^1	Π^2	Π^3
1	b_2	b_2	b_2
2	a_1	a_1	a_1
3	c_4	c_4	c_4
4	b_3	b_3	b_3
5	a_6	a_6	a_6
6	c_5	c_5	c_5
7	a_7	c_{11}	c_{11}
8	d_8	d_8	d_8
9	d_9	d_9	d_9
10	c_{11}	a_7	a_7
11	–	–	–
12	c_{16}	b_{12}	c_{16}
13	d_{13}	d_{13}	d_{13}
14	d_{14}	d_{14}	d_{14}
15	b_{12}	c_{16}	b_{12}
16	–	–	–
A	+1	+0.1	+0.1
B	+0.1	+1	+0.1
C	+0.75	+0.75	+1.8

(a)

Slot	Π^4	Π^5
1	b_2	b_2
2	a_1	a_1
3	c_4	c_4
4	b_3	b_3
5	a_6	a_6
6	c_5	c_5
7	a_7	c_{11}
8	d_8	d_8
9	d_9	d_9
10	(destroyed)	a_7
11	c_{11}	–
12	c_{16}	b_{12}
13	d_{13}	d_{13}
14	d_{14}	d_{14}
15	b_{12}	(destroyed)
16	–	c_{16}
A	+1	+0.1
B	+0.1	+1
C	+0.4	+0.4

(b)

Table 2: (a) The three possible schedules chosen by an *individual rational*, *Pareto-efficient* rule. They differ only in slots 7, 10, 12, and 15. (b) Two landing schedules following slot destruction in I.

is only one landing schedule for $I_{S\to S\setminus\{10\}}$ that satisfies the conditions for *Pareto-efficiency* and *individual rationality*. It is Π^4 described in Table 2b.

Therefore $\varphi(I_{S\to S\setminus\{10\}}) = \Pi^4$. However, A's gain of one unit is greater than his gain at Π^2 or Π^3 above, i.e. $\varphi_A(I_{S\to S\setminus\{10\}}) \succ^w_A \varphi_A(I)$. Since airline A benefits from destroying its vacant slot 10, φ is manipulable by slot destruction in this case.

Therefore we must have $\varphi(I) = \Pi^1$. Similar to the argument above, let $I_{S\to S\setminus\{15\}}$ be the instance obtained from I when airline B destroys slot 15. There is only one landing schedule for $I_{S\to S\setminus\{15\}}$, namely Π^5, that satisfies the conditions for *Pareto-efficiency* and *individual rationality*.

Therefore $\varphi(I_{S\to S\setminus\{15\}}) = \Pi^5$. However, B's gain of one unit is greater than its gain at Π^1 above, i.e. $\varphi_B(I_{S\to S\setminus\{15\}}) \succ^w_B \varphi_B(I)$. Since airline B benefits from destroying its vacant slot 15, φ is manipulable by slot destruction. □

5 FAA-conforming Rules

The previous section shows that if an *individually rational* reassignment rule is *Pareto-efficient*, then it is vulnerable to each of three separate kinds of manipulation. In this section, we show that some degree of non-manipulability can be recovered by replacing *Pareto-efficiency* with the more modest goal of *non-wastefulness*. In fact we provide a single class of rules each of which *simultaneously* satisfies two of our incentives properties. Furthermore, this rule is simple—in a sense to be defined below—implying that *part* of its computation can be decentralized to the airlines in precisely the same way that the FAA's current reassignment rule does. Finally, the class of rules satisfies a basic procedural property right given to the airlines by the FAA.

In this section, we begin by motivating our definition of *FAA-conforming rules*. Such rules achieve the basic objective of *non-wastefulness*, satisfy the property right of *self-optimization* defined below, and use a restricted set of information about instances (*simple*, defined below). We show that, unfortunately, such rules *must* be manipulable by arrival times. We then provide the class of rules—based on deferred acceptance—that satisfies the other incentives requirements.

5.1 FAA Conformation

One can think of two main ways in which flight weight information (w) can be used to improve efficiency: to make both inter- and intra-airline "trades." By comparing relative flight weight ratios across different airlines, one can find Pareto-improving (inter-airline) trades. Unfortunately, Theorems 1–3 show that the use of weights to execute *all* such efficient trades conflicts with non-manipulability. By comparing weights within a single airline's schedule, the airline can optimally arrange its own flights *within its own portion* of the landing schedule. As we shall see, this second use of weight information need not conflict with two of our three non-manipulability conditions.

Of course a reassignment rule could completely ignore flight weight information altogether. Consider such a rule operating on the following trivial instance.

Slot	Flight	Airline	Earliest	Weight
1		B		
2	a_2	A	1	w_2
3	a_3	A	1	w_3

A *non-wasteful* rule assigns the flights to the first two slots. If the rule ignores weight information and assigns a_2 to slot 1, then the rule can be manipulated (when $w_2 < w_3$) by misreporting $e'_{a_2} = 2$. It is similarly manipulable (when $w_2 > w_3$) if it assigns a_3 to slot 1.

This example makes a simple point about manipulability in environments where agents (airlines) are obligated to follow the landing schedule posted by the reassignment rule. On the other hand it may not be convincing if one presumes that airline A should have the right to swap flights a_2 and a_3 as it wishes. Indeed this is the case in the application to the U.S. F.A.A., where airlines are granted this right by law. In such settings, it is clear that, after a reassignment rule operates, each airline will rearrange its own part of the schedule in an ideal way. We formalize this by calling an assignment *self-optimized* when each airline is using *its own slots* in the best way possible.

Definition 8. An assignment (Π, Φ) is **self-optimized** (for instance I) if there exists no airline A and no landing schedule Π' such that both (i) $\Pi' \succ_A^w \Pi$ and (ii) $\Pi'(f) \in \Phi(A)$ for all $f \in F_A$. Given any ownership function Φ (that can yield a consistent, feasible landing schedule), denote by $\boldsymbol{SO(\Phi, I)}$ the set of landing schedules Π such that (Π, Φ) is self-optimized for I.

We also say that a landing schedule Π is *self-optimized* if it is part of a self-optimized assignment (Π, Φ) for some Φ. A reassignment rule φ is *self-optimized* if it always outputs a *self-optimized* landing schedule (with respect to the reported parameters).

Our motivation for this condition is strong: when each agent is given the procedural right to rearrange its own part of the schedule (as with the FAA), it is without loss of generality to restrict attention to reassignment rules that output *self-optimized* landing schedules. This follows from a revelation principle type of argument.[25]

A second attribute of rules used by the FAA regards not only the *use* but in fact the *communication* of weight information. The FAA does not directly solicit information from airlines about their willingness to make tradeoffs.[26] Of course, airlines implicitly use such information when they create self-optimized portions of their landing schedules. Therefore, in practice, weight information is used *only* to self-optimize. Phrased differently, weight information cannot impact the allocation of slots to *airlines*, though it can impact the assignment of any single airline's *flights* to its slots. Below we define such rules as *simple*.[27]

Definition 9. A reassignment rule is **simple** if for any instances I and I' with weight profiles w and w', if $I' = I_{w \to w'}$ then for all $A \in \mathcal{A}$, $\varphi_A(I) = \varphi_A(I')$. In words, if an instance is altered only by a change of weights, then no airline consumes a different set of slots.[28]

To be clear, there are two separate motivations for this restriction on rules. The first is the applied motivation given above, regarding the FAA's current procedures. The need to report preference information can increase complexity (and hence operational costs) for the airlines. The FAA currently uses a *simple* rule (the Compression Algorithm), and Schummer and Vohra (2012) propose an alternative *simple* rule based on the Top Trading Cycle algorithm.

A second (partial) theoretical motivation involves the negative results of Section 4, showing that weight information cannot be used to achieve full efficiency in the presence of any of our incentive constraints.[29]

Finally, since the FAA's most basic of motivations is to avoid slots going to waste, the following definition is justified by the arguments above.

[25] The argument is as follows. Suppose that a rule assigns slots to airlines and that each airline may reassign its assigned slots amongst its own flights. Further suppose that, conditional on this reassignment right, each airline has a dominant strategy of truthfully reporting preference information (e.g. e's or w's) to the rule. Then, such incentives would be preserved if the airline were forced to communicate its preferences to a trustworthy proxy who is responsible for both (i) reporting the airline's preferences to the rule and (ii) afterwards, optimally reassigning that airline's slots to its flights. Hence in our analysis of rules satisfying such incentive compatibility conditions, it is without loss of generality to suppose that the rule chooses self-optimized outcomes on behalf of the airlines, based on their reported preference information.

[26] One exception to this statement is the Slot Credit Substitution procedure, which has been described (Robyn (2007)) as unwieldy; see the discussion in Schummer and Vohra (2012).

[27] A more accurate yet tedious term could be "semi-weight-invariant" rules.

[28] A stronger version of this definition would require $\varphi(I) = \varphi(I')$, i.e. each flight to be assigned the same slot. Such rules cannot be self-optimized.

[29] A middle ground is to search for incentive compatible rules that capture *some* but not all efficiency from trade based on w. Such rules would be more complex than current FAA procedures, but is still an interesting question we leave to future research.

Definition 10. A reassignment rule is **FAA-conforming** if it is *non-wasteful*, *simple*, and *self-optimizing*.

5.2 Manipulability through arrival times

Since *simplicity* is a form of invariance with respect to the airlines' reported weights w, it is not surprising that it essentially rules out w-manipulability. If a rule is *simple* then either it is non-manipulable by weights, or it can be used to trivially construct another simple rule that is Pareto-dominating.

Observation. Let φ be a *simple* reassignment rule. Define φ' to be a *self-optimized* rule that satisfies $\{\varphi_f(I) : f \in F_A\} = \{\varphi_f(I) : f \in F_A\}$ for each I and for each $A \in \mathcal{A}$. Then φ' is both *simple* and not *manipulable by weights*.

The proof of this is obvious. What is less obvious, however, is whether such a rule can also be immune to other forms of manipulation: misreporting earliest arrival times or by slot destruction. Our answers to these two questions are mixed. No such rule can avoid the former, which we now show. The good news, however, is that a class of simple rules *is* immune to manipulation by slot destruction (and more), as we show in Subsection 5.3.

Theorem 4. *If an* FAA-conforming *reassignment rule is* individually rational*, then it is* manipulable via earliest arrival times*.*

Proof. Consider instances I and I' defined by weights w and w' in the following table.

Slot	Flight	Airline	Earliest e	Weight w	Weight w'
1	–	A			
2	–	A			
3	b_3	B	1	1	1
4	c_4	C	1	1	1
5	b_5	B	2	5	1
6	c_6	C	3	5	5
7	b_7	B	4	1	7

Let φ be a *simple*, *non-wasteful* reassignment rule. Let $\varphi_B(I)$ denote the three of those slots assigned to B's flights, i.e. $\varphi_B(I) = \{\varphi_{b_3}(I), \varphi_{b_5}(I), \varphi_{b_7}(I)\}$. Since φ is *non-wasteful*, it must assign all five flights to slots 1–5, so $\varphi_B(I) \subset \{1, 2, 3, 4, 5\}$. Define $\varphi_C(I)$ and $\varphi_B(I')$ similarly. Since φ is *simple*, note that $\varphi_B(I') = \varphi_B(I)$ (although $\varphi_f(I) \neq \varphi_f(I')$ is certainly possible).

There are $\binom{5}{3} = 10$ candidate subsets to consider for $\varphi_B(I)$, but B's flights cannot feasibly be assigned to $\{1, 2, 3\}$ or $\{3, 4, 5\}$. We show that for each of the remaining eight possibilities, φ must be e-manipulable in some way.

Case 1: $\varphi_B(I)$ is $\{1, 3, 4\}$, $\{1, 3, 5\}$, or $\{1, 4, 5\}$.

In this case $\varphi_{b_5}(I) \geq 3$. From instance I, consider airline B misrepresenting e_{b_3} to be $e'_{b_3} = 2$ (resulting in $\tilde{I} = I_{e_{b_3} \to e'_{b_3}}$). By *non-wastefulness*, $\varphi_{c_4}(\tilde{I}) = 1$. With *simplicity* (and *self-optimization*) this yields $\varphi_{b_5}(\tilde{I}) = 2$. This improvement for b_5 gives B a relative gain of at least 5 (weight) units. It is simple to check that B can lose at most 4 units combined through changes in the assignment of b_3 and b_7. Since B would gain from such a manipulation, this rules out Case 1. In all remaining cases, B must receive slot 2.

Case 2: $\varphi_B(I)$ is $\{2, 3, 4\}$ or $\{2, 3, 5\}$.

In this case $\varphi_C(I)$ is $\{1, 5\}$ or $\{1, 4\}$, so $\varphi_{c_4}(I) = 1$ and $\varphi_{c_6}(I) \geq 4$. From instance I, consider airline C misrepresenting e_{c_4} to be $e'_{c_4} = 3$ (resulting in $\tilde{I} = I_{e_{c_4} \to e'_{c_4}}$). By *non-wastefulness* B receives the first two slots, and either $\varphi_{c_4}(\tilde{I}) = 3$ or $\varphi_{c_6}(\tilde{I}) = 3$, with *self-optimization* implying the latter. This improvement for c_6 gives C a relative gain of at least 5 (weight) units, while C can lose at most 4 units through the change in the assignment of c_6. Since C would gain from such a manipulation, this rules out Case 2. In all remaining cases, C must receive slot 3.

Case 3: $\varphi_B(I) = \{2, 4, 5\}$.

In this case $\varphi_{b_5}(I) = 2$, while b_3 and b_7 go to slots 4 and 5 (the order being irrelevant). From instance I, consider airline B misrepresenting e_{b_3} to be $e'_{b_3} = 3$ (resulting in $\tilde{I} = I_{e_{b_3} \to e'_{b_3}}$). By *non-wastefulness*, $\varphi_{c_4}(\tilde{I}) = 1$ and $\varphi_{b_5}(\tilde{I}) = 2$.

We also show $\varphi_{b_3}(\tilde{I}) = 3$. Suppose not, so $\varphi_{b_3}(\tilde{I}) > 3$. Consider yet another instance I'', obtained from $\tilde{I}$ by giving sufficiently high weight w''_{b_3} to b_3. By *simplicity*, B's flights would be allocated the same set of slots as in $\tilde{I}$, and in particular B would not be assigned slot 3 at I''. Since this implies $\varphi_{b_3}(I'') > 3$, $\varphi(I'')$ would violate *individual rationality* for B. Therefore $\varphi_{b_3}(\tilde{I}) = 3$.

But then at worst $\varphi_B(\tilde{I}) = \{2, 3, 5\}$, i.e. $\varphi_{b_5}(I) = \varphi_{b_5}(\tilde{I})$, while B gains via the other two flights. Since B would gain from such a manipulation, this rules out Case 3. In the two remaining cases, B must receive slot 1.

Case 4: $\varphi_B(I) = \{1, 2, 5\}$.

Recall that *simplicity* then implies $\varphi_B(I') = \{1, 2, 5\}$, so $\varphi_{b_7}(I') = 5$. From instance I', consider airline B misrepresenting e_{b_5} to be $e'_{b_5} = 4$ (resulting in $\tilde{I} = I'_{e_{b_5} \to e'_{b_5}}$). By *non-wastefulness*, flights c_4 and b_3 are assigned the first two slots (in some order) and $\varphi_{c_6}(\tilde{I}) = 3$. As B receives slots 4 and 5, *self-optimization* results in $\varphi_{b_7}(\tilde{I}) = 4$. This improvement for

b_7 gives B a relative gain of 7 units, while B must lose strictly fewer than that through the change in the assignments of b_3 and b_5. This rules out Case 4.
Case 5: $\varphi_B(I) = \{1, 2, 4\}$ (so $\varphi_C(I) = \{3, 5\}$).

By self-optimization, $\varphi_{c_6}(I) = 3$. From instance I, consider airline C misrepresenting e_{c_4} to be $e'_{c_4} = 4$ (resulting in $\tilde{I} = I_{e_{c_4} \to e'_{c_4}}$). By *non-wastefulness*, $\varphi_{b_3}(\tilde{I}) = 1$, $\varphi_{b_5}(\tilde{I}) = 2$, and $\varphi_{c_6}(\tilde{I}) = 3$.

We also show $\varphi_{c_4}(\tilde{I}) = 4$. Suppose not, so $\varphi_{c_4}(\tilde{I}) = 5$. Consider yet another instance I''' obtained from $\tilde{I}$ by giving sufficiently high weight w'''_{c_4} to c_4. By *simplicity*, C's flights would be allocated the same set of slots as in $\tilde{I}$, and in particular C would not be assigned slot 4. Since this implies $\varphi_{c_4}(I''') = 5$, $\varphi(I''')$ would violate *individual rationality* for C. Therefore $\varphi_{c_4}(\tilde{I}) = 4$.

Since $\varphi_{c_6}(\tilde{I}) = \varphi_{c_6}(I)$ and $e_{c_4} \leq \varphi_{c_4}(\tilde{I}) < \varphi_{c_4}(I)$, φ is *e-manipulable*, completing the proof. □

While no reasonable *simple* rule can avoid e-manipulability, there are weaker forms of incentive compatibility that can be achieved. Schummer and Vohra (2012) exhibit two rules that cannot be manipulated—by an airline misreporting e_f's—in such a way as to benefit *all* flights of that airline. That is, if any flight of an airline gains through such a misreport, another flight must move to a worse slot. One of those rules is the one currently used by the FAA, while the other is based on the classic Top Trading Cycles algorithm. We consider this condition in Section 6

5.3 Deferred Acceptance with Self Optimization

In order to define rules based on deferred-acceptance, we need to introduce two preliminary concepts: airline *choice functions* over sets of slots, and the slots' *priority orders* over airlines. We begin with the former, which slightly extends the concepts in Roth (1984) to our environment, where each airline cares about how its flights are assigned within the set of slots it receives.

5.3.1 Choice sets and priorities

Firstly, to define choice functions, consider an airline $A \in \mathcal{A}$ with flights F_A and preferences $\succ^w_A$. Given a set of slots $T \subseteq S$, how would A choose to assign its flights within T? Assuming it can feasibly do so, determining A's "self-optimal" assignment of F_A to T typically requires knowing weights w_f, $f \in F_A$. It is simple to see, however, that even without weight information, one can determine the *subset* of T that A would choose to occupy. Clearly A

would not want to assigning some $f \in F_A$ to a slot t while leaving vacant some slot s with $e_f \leq s < t$. This necessary condition is sufficient to identify the unique subset of T that A would choose to occupy, which allows us to define choice functions as follows.

Definition 11. Fix an instance I, airline A, and set $T \subseteq S$ such that A's flights can feasibly be scheduled within T. Airline A's **choice function** $C_A(T)$ over such sets $T \subseteq S$ is the output of the following simple algorithm.

- Order flights in F_A in increasing order of e_f (break ties arbitrarily).
- Assign flights sequentially to the earliest slot in T that each flight can feasibly use.
- Denote the set of occupied slots $C_A(T) \subseteq T$.

It is straightforward to see that if an airline A could assign its flights (self-optimally) within $T \subseteq S$, then its flights would occupy $C_A(T)$.[30] It turns out that choice functions in this environment satisfy the classic substitutability condition; see Kelso and Crawford (1982) and Roth (1984).

Secondly, to define priority orderings that parameterize deferred-acceptance-based rules in our model, it is simplest to think of the set of airlines $\mathcal{A}$ as being fixed.[31] Let $\mathcal{I}(\mathcal{A})$ denote the domain of instances in which $\mathcal{A}$ is the existing set of airlines. For any positive integer (interpreted as a potential slot) $s \in \{1, 2, \ldots\}$, a **priority order** (on $\mathcal{A}$), $\gg_s$, is simply a linear order over the airlines in $\mathcal{A}$. For an instance $I \in \mathcal{I}(\mathcal{A})$, $(\gg_s)_{s \in S}$ is a **profile of priority orders**.

5.3.2 Defining DASO Rules

For a fixed set of airlines $\mathcal{A}$ and profile of priorities $(\gg_s)$, we define a rule on the domain $\mathcal{I}(\mathcal{A})$. The rule is based on the deferred acceptance algorithm, augmented with a self-optimization step. Recall from Definition 8 that $SO(\Phi, I)$ is the set of self-optimized landing schedules for I with respect to slot ownership function Φ.

Definition 12 (DASO rules)**.** For a set of airlines $\mathcal{A}$ and profile of priorities $(\gg_s)$ on $\mathcal{A}$, the **deferred acceptance with self-optimization (DASO) rule (with respect to $\gg$)** associates with every instance $I \in \mathcal{I}(\mathcal{A})$ the landing schedule computed with the following "DASO algorithm."

[30] Of course the flights may not be assigned as in this algorithm, as the airline may wish to swap the positions of two or more flights.

[31] It becomes straightforward to generalize the definition to a domain in which this set varies.

Step 0: Each slot s *proposes* to the airline A that owns it ($s \in \Phi^0(A)$). Let T_A^0 denote the slots who proposed to any $A \in \mathcal{A}$.[32] For each $A \in \mathcal{A}$, determine $C_A(T_A^0)$. We say that A *rejects* each slot $s \in T_A^0 \setminus C_A(T_A^0)$. If there are no rejected slots, proceed to the Self-optimization step.

Step $k = 1, \ldots$: Each slot s that was rejected in step $k-1$ proposes to the highest-ranked airline in $\gg_s$ that has not already rejected s in some earlier step. (If no such airline exists, s is to be permanently unassigned.) Let T_A^k denote the slots who proposed to A in step k *plus* those in $C_A(T_A^{k-1})$. For each airline A, determine $C_A(T_A^k)$. We say that A *rejects* each slot $s \in T_A^k \setminus C_A(T_A^k)$. If there are no rejected slots, proceed to the Self-optimization step.

Self-optimization step: For each airline A, assign A's flights to the last $C_A(T_A^k)$ so that the resulting landing schedule is self-optimized. Break ties among equally-weighted flights by preserving their relative order in Π^0.[33]

The DASO algorithm is simply the well-known algorithm of Gale and Shapley (1962) with two adjustments: an instance-specific adjustment of priorities in Step 0, and the addition of a self-optimization step. It is easily seen to end in a finite number of steps based on the known finiteness of (classic) deferred acceptance. Some further remarks are in order.

First, having slots propose to owners in Step 0 plays two related roles. It guarantees the final landing schedule to satisfy *individual rationality*, and guarantees each airline to have a feasible set of slots to choose form in every step. (E.g. if no slot proposed to A in the first step, then A would not have feasible sets to choose from.)

Secondly, we have described what could be called the "slot proposing" version of the deferred acceptance algorithm. An alternate "airline proposing" version of the algorithm would have airlines proposing to their favorite sets of slots, and slots accepting proposals only from their most-preferred airline. Indeed such a formulation would seem more appropriate to readers familiar with the matching literature, since such an algorithm would tend to result in better outcomes for the airlines.[34] In this model, however, it turns out that *both versions of the algorithm would produce precisely the same matching.*

Observation. Fixing priorities and an instance, an airline-proposing version of the DASO

[32] In fact in Step 0, $T_A^0 = \Phi^0(A)$. The notation is not redundant is later steps.

[33] This tie-breaking is irrelevant other than to simplify exposition in proofs and to simplify a welfare statement in Theorem 6.

[34] An even more basic question is whether such an algorithm would be well-defined. It can be easily shown (see online appendix) that airline preferences in our model satisfy the classic substitutability condition, which Roth (1984) invokes to compare college- and student-optimal stable outcomes.

algorithm would yield precisely the same outcome as the slot-proposing version described above.

This general statement requires a proof (see online appendix), though it follows from an intuitive induction argument.[35] In terms of exposition, however, we have stated DASO using the slot-proposing description, which makes our proofs easier to state.[36]

5.3.3 Results

We begin with the straightforward observations that a DASO rule satisfies the following three properties.

Theorem 5. *For any profile of priorities, the corresponding DASO rule is* non-manipulable via weights*,* individually rational*, and* non-wasteful*.*

Proof. The algorithm makes no use of weights information until the Self-optimizing step, by which time the set of slots to be received by any airline $A \in \mathcal{A}$ has been fully determined. Subject to the constraint that A receives precisely that set of slots, the Self-optimization step uses weights only to give A its most-preferred landing schedule. Therefore it is obviously in an airline's best interests to report this information truthfully.

In Step 0 of the algorithm, each airline chooses its favorite set of slots from amongst those which it owns (under the implicit assumption that it may assign flights to the set in a self-optimized way). At each subsequent step, an airline either keeps its current set or selects a better one. Hence the rule is *individually rational*. It is obvious that the rule is *non-wasteful*. □

Remark 2. DASO rules satisfy the stronger version of *individual rationality* discussed in Subsection 3.2. Namely, a DASO rule yields an outcome that any airline A (weakly) prefers to what A could achieve by only rearranging its flights within $\Phi^0(A)$.

The following lemma helps us prove the main result of this section. It states that an airline A never "regrets" rejecting a slot s at some step of the algorithm, in the sense that if the algorithm later assigns A's flights to some set of slots T, then A could not benefit

[35] Specifically, suppose that the choice of proposers is irrelevant in allocating the first $s-1$ slots to airlines. Then of the remaining slots, slot s is considered "best" (in a well-defined sense due to substitutability) by all of the airlines that can still use s (subject to the allocation of the first $s-1$ slots). Therefore the highest-ranked such airline will receive s regardless of who is doing the proposing.

[36] Another property specific to the slot-proposing version is that each step of the algorithm yields a feasible landing schedule.

by replacing some $s' \in T$ with s.[37] This result is similar to Proposition 3 ("Rejections are final") in Roth (1984).

Lemma 1. *Fix an instance I and a DASO rule φ. Suppose airline $A \in \mathcal{A}$ and slot $s \in S$ satisfy $s \in T_A^k \setminus C_A(T_A^k)$ (s was rejected by A) in some step k of the DASO algorithm. Then for all steps $\ell > k$ of the algorithm, $s \notin C_A(T_A^\ell \cup \{s\})$. In particular, for all $f \in F_A$, $e_f \leq s$ implies $\varphi_f(I) < s$.*

Proof. Denote the flights of A that could feasibly use s as $G_A = \{f \in F_A : e_f \leq s\}$. If A rejects s in step k ($s \notin C_A(T_A^k)$), then T_A^k contains at least $|G_A|$ slots strictly earlier than s (see the algorithm in Definition 11).

This implies that $C_A(T_A^k)$ contains $|G_A|$ slots strictly earlier than s. At every step of the algorithm $\ell \geq k$, if $C_A(T_A^\ell)$ contains at least $|G_A|$ such slots, then so do $T_A^{\ell+1}$ and $C_A(T_A^{\ell+1})$. Furthermore, at the conclusion of the algorithm, each $f \in G_A$ satisfies $\varphi_f(I) < s$. □

Our main positive result is that DASO rules induce airlines to truthfully report flight cancelations. Not only are they immune to *manipulation via slot destruction* as defined earlier, but an airline cannot even manipulate by "hiding" a vacant slot from the rule and later consuming it (even in a self-optimized way). We formalize this as follows.

Definition 13. A reallocation rule φ is **manipulable via slot hiding** if there is an instance $I = (S, \mathcal{A}, (F_A)_{A \in \mathcal{A}}, e, w, \Pi^0, \Phi^0)$, airline $A \in \mathcal{A}$, and slot $s \in S$ such that

(i) $s \in \Phi^0(A)$ (A owns s),

(ii) $\nexists f \in F$ such that $\Pi^0(f) = s$ (s is initially vacant), and

(iii) $\exists \Pi$ such that $\Pi \succ_A^w \varphi(I)$ and $\bigcup_{F_A} \Pi(f) \subset \left(\bigcup_{F_A} \varphi_f(I_{S \to S \setminus \{s\}})\right) \cup \{s\}$.

Without loss of generality one can interpret Π in (iii) to be a self-optimized landing schedule when A has access to the slots he is allocated at $I_{S \to S \setminus \{s\}}$ plus s.

DASO rules are not *manipulable via slot hiding*, but we prove an even stronger result: if an airline attempts to manipulate by hiding a slot, then *no* airline can be made better off. Such a strong result is important in environments where group incentives are relevant. Even if one airline had the ability to somehow "compensate" another airline to postpone the

[37] One should be careful here: an airline could regret not having rejected a slot, in the sense that if it had, then it would have received a better set of slots later. Thus this is a limited no-regret result: a truthfully rejected slot later must be considered useless by the airline.

announcement of a flight cancelation, our result shows that such an arrangement cannot be beneficial.[38]

To be clear what the result states, given a set of slots S we are taking a priorities $(\gg_s)_{s\in S}$ as given. After supposing that an airline A hides some slot $h \in S$, we make a type of consistency assumption that the DASO rule simply uses the set of priorities $(\gg_s)_{s\in S\setminus\{h\}}$ to compute the outcome for the new instance (without h). We are not allowing the priorities to vary with respect to the set of candidate slots.[39]

Theorem 6. *No DASO rule is* manipulable via slot hiding. *In fact under any DASO rule, if an airline hides a slot, then* all *airlines (weakly) worse off.*

Proof. Suppose by contradiction that there is an instance I, airline $B \in \mathcal{A}$, and slot $h \in \Phi(B)$ such that some airline A gains when B hides h. Let $I' = I_{S\to S\setminus\{s\}}$ be the instance obtained from I by deleting slot h. Denote by Π the landing schedule output by $\text{DASO}(\gg_s)_S$ for I; denote by Π' the landing schedule output by $\text{DASO}(\gg_s)_{S\setminus t}$ for I'. Finally, let Π'' be the landing schedule obtained from Π' by "unhiding h," i.e. by self-optimizing the flights of airline B over the slots $C_B(\Pi'(F_B) \cup \{s\})$.

To simplify exposition, assume that the owner of any slot s has the highest priority for that slot under $\gg_s$. (This is without loss of generality due to the first step of DASO, in which slots first propose to their owners.)

Since A gains by the hiding of h, there must exist some $f \in F_A$ such that $\Pi''(f) < \Pi(f)$. Without loss of generality, suppose that $f \in F_A$ is the *earliest* flight that receives an earlier slot under Π'' than under Π, i.e. let $f \in F_A$ be such that both $\Pi''(f) < \Pi(f)$ and

$$\forall f' \in F \setminus \{f\}, \qquad \Pi''(f') < \Pi(f') \Longrightarrow \Pi''(f') > \Pi''(f). \tag{1}$$

Denote $s' = \Pi''(f)$.

Since DASO rules are *non-wastefulness*, s' must be occupied by some $f' \neq f$ under Π (otherwise it could be given to f); i.e. there exist $A' \in \mathcal{A}$ and $f' \in F_{A'}$ such that $\Pi(f') = s'$. Airline A' may or may not coincide with A or B.

Case 1: $A' = A$. Both flights $f, f' \in F_A$ can feasibly use s', but $s' = \Pi(f') < \Pi(f)$. Therefore the *Self-optimization* step of DASO (applied to I) implies that either $w_{f'} > w_f$, or both $w_{f'} = w_f$ and $\Pi^0(f') < \Pi^0(f)$ (the tie-breaking condition). In either case, since

[38] This kind of possibility result for such group incentives contrasts with impossibility results on more general domains such as Schummer (2000).

[39] One can imagine generalizing DASO rules to let $\gg$ be a function of various parameters, as we discuss in Subsection 5.4. The Theorem might not hold for such generalizations.

$\Pi''(f) = s'$, this implies $\Pi''(f') < \Pi''(f)$ (because Π'' is self-optimized). This contradicts eqn. (Equation 1), the assumption that f was the earliest flight to move up in Π''.

Case 2: $A' \neq A$. Observe that at Π, A would strictly gain by receiving s'. That is, A does not receive s' under Π, but desires s' in that both $e_f \leq s'$ and $\Pi(f) > s'$. Lemma 1 thus implies that s' never proposed to A at any step of the DASO algorithm under I. Since A' receives s' under I (hence s' *did* propose to A'), s' puts higher priority on A' than on A: $A' \gg_{s'} A$.

If $h = s'$ then we have $A = B$, and thus $A \gg_{s'} A'$ (from the assumption on $\gg$ above). Since this is a contradiction, we must have $h \neq s'$. This implies that A receives s' under Π'.

In order for A to receive s' when DASO is applied to I', s' must propose to A, implying that A' must have rejected a proposal from s' ($A' \gg_{s'} A$) at some step of the algorithm. Lemma 1 thus implies that A' cannot strictly gain at Π'' from receiving s', hence $\Pi'(f') < s' = \Pi(f')$. This contradicts eqn. (1).

Since both cases lead to a contradiction, no flight can move to an earlier slot following the hiding of any vacant slot. □

Theorem 6 is related to a result of Crawford (1991) in the generalized College Admissions model with contracts of Roth (1984). Under assumptions also satisfied in our model, Crawford shows that adding a student to an economy benefits all colleges when implementing either the student-optimal or college-optimal stable outcome.[40] Applied to our model, this can be shown to imply that if a slot is *destroyed*, then all airlines would be (weakly) worse off under a DASO rule. Our Theorem strengthens this conclusion: an airline cannot gain even by temporarily *hiding* a vacant slot and later consuming it.

5.4 Generalizing DASO

The DASO rules described above are defined with respect to a fixed list of priority orderings ($\gg_s$) of the slots. This concept can be generalized by allowing the priorities to be a function of various parameters of $I = (S, \mathcal{A}, (F_A)_{A\in\mathcal{A}}, e, w, \Pi^0, \Phi^0)$. For instance, after a rule asks agents to report their e_f's and w_f's, a priority order could be determined as a function of those parameters. DASO rules as described earlier would be a special case in which, of all parameters of I, priorities depend only on $\mathcal{A}$.

Intuitively it seems clear that such generalizations allow for more possibilities of manipulation. Perhaps by misreporting some w_f, an airline could obtain a higher priority for some slot, making the rule w-manipulable in contrast to Theorem 5.

[40] See also Kelso and Crawford (1982) and Kojima and Manea (2010).

However, by expanding the set of rules, one might hope to achieve a positive result regarding e-manipulability. After all, Theorem 4 does not apply to such generalizations since, by allowing $\gg$ to depend on w, the rule would no longer be *simple*. A basic example demonstrates, however, that all natural generalizations of DASO rules also are e-manipulable.

Slot	Flight	Airline	Earliest	Weight
1	$\emptyset$	A		
2	b_2	B	1	1
3	a_3	A	1	1
4	a_4	A	2	5

Regardless of the structure of $\gg$, a DASO-based rule would ultimately assign a_3 to slot 1. *Individual rationality* then implies that b_2 remains in slot 2, and a_4 moves up to slot 3. However, by misreporting $e'_{a_3} = 3$, a DASO rule—in fact any nonwasteful, self-optimized rule—moves a_4 up to slot 2, which yields an improvement for A. The intuition in this example is that A can "force" a trade my misreporting. On the other hand, a deferred-acceptance-based rule—at least of the kind we propose here—that is designed to respect individual rationality must give an airline highest priority among the slots it initially owns under Φ^0.

6 Weak Incentives and Self-optimization

The classic *strategy-proofness* condition makes it a dominant strategy for agents to truthfully reveal *all* of their preference information. Our three incentive compatibility conditions weaken this concept in that each one requires this dominance only for a specific type of preference (or endowment) information, namely information pertaining to flight arrival times, relative costs of flight delays, or flight cancelations. Despite this weakening, the results of Section 4 and Theorem 4 show that these three conditions can be somewhat demanding.

Hence we weaken our conditions by eliminating only situations in which an airline can manipulate in order to improve the positions of *each* of its flights. Such a condition was introduced to this class of problems by Schummer and Vohra (2012) for the case of misreporting arrival times. They show that such a condition is satisfied by the two reallocation rules they consider, as we discuss further below. We show that DASO rules also satisfy this condition. The definition states that there should exist no misreport of arrival time that strictly benefits one of A's flights without hurting any of its other flights.

Definition 14. A reallocation rule φ is **weakly non-manipulable via earliest arrival times** if there is no instance $I = (S, \mathcal{A}, (F_A)_{A\in\mathcal{A}}, e, w, \Pi^0, \Phi^0)$, airline $A \in \mathcal{A}$, flight $f \in F_A$, and an earliest arrival time e'_f such that, letting e' be obtained from e by replacing e_f with e'_f,

$$\forall f \in F_A \quad \varphi_f(I_{e\to e'}) \leq \varphi_f(I)$$

with strict inequality for at least one $f \in F_A$.

Recall that if a rule is *individually rational* and *FAA-conforming*, then it is *manipulable via earliest arrival times* (Theorem 4). The following possibility result is obtained by weakening the incentives condition.

Proposition 7. *DASO is* weakly non-manipulable via earliest arrival times*.*

Proof. Fix a DASO rule corresponding to some priority ordering ($\gg$). Fix an instance I, airline $B \in \mathcal{A}$, and flight $g \in F_B$, and let Π be the outcome of the DASO rule for I.

Let $I' = I_{e_g \to e'_g}$ be the instance in which B misreports e_g to be e'_g, and let Π' be the outcome of the DASO rule for I'. Suppose by contradiction that for all $f \in F_B$, $e_f \leq \Pi'(f) \leq \Pi(f)$, and that $\Pi'(f) < \Pi(f)$ for at least one such flight.

Since there is at least one slot that is assigned to distinct airlines under Π and Π', denote by $s \in S$ the earliest such slot. Namely, $s = \Pi(f) = \Pi'(f')$ for some $f \in F_A$ and $f' \in F_{A'}$, where $A \neq A'$, and s is the earliest such slot.

Consider the number of slots airline A obtains at time s or earlier. By our choice of s, A receives fewer such slots at Π' than at Π. Therefore, $B \neq A$.

Case 1: $A' \neq B$**.** Note that since $A \neq B \neq A'$, both A and A' report the same arrival times at both I and I'.

Recall that under Π and Π', the airlines receive the same sets of slots strictly earlier than s. Since A receives s under Π, A could be made better off at Π' by being offered s. Therefore (following Lemma 1) s never proposed to A in the DASO algorithm under I', hence $A' \gg_s A$.

Symmetrically, since A' receives s under Π', A' could be made better off at Π by being offered s. The same arguments yield $A \gg_s A'$, which is a contradiction.

Case 2: $A' = B$**.** As above, $B \neq A$ implies $A' \gg_s A$ (which in this case means $B \gg_s A$).

Since A receives s at Π, $A' = B$ must have rejected s at some stage of the DASO algorithm applied to I. Lemma 1 therefore implies that B cannot gain by receiving s at Π; in other words, all of B's flights that can feasibly use s must be assigned to even earlier slots under Π. Since B receives the same number of such earlier slots at Π', B cannot feasibly use s according to the arrival times e reported at I. Therefore, B must have misreported an infeasible e'_g

causing it to receive a slot s that it cannot feasibly use under its true arrival times. This contradicts the assumption that airline $B = A'$ benefits from such a misreport. □

Schummer and Vohra (2012) consider two other simple rules: the *Compression* algorithm currently used by the FAA and an adaptation of Shapley and Scarf's (1974) Top Trading Cycle algorithm which we denote *TC*. Without allowing for self-optimization, they show that both of these simple rules also are weakly non-manipulable via earliest arrival times. However both of those rules, like DASO rules, fail the stronger incentives condition, i.e. are *manipulable via arrival times*.[41]

It turns out that the two weak incentives results of Schummer and Vohra (2012) are robust to the self-optimization of the initial landing schedule.

Observation. Schummer and Vohra (2012) show that both the Compression algorithm and their TradeCycle rule are weakly non-manipulable via earliest arrival times. If either rule were augmented by first self-optimizing the initial landing schedule, then it would remain weakly non-manipulable.

For space reasons we omit proofs.[42]

On the other hand, it turns out that if either rule were augmented by only self-optimizing the resulting landing schedule (and not necessarily the initial one), then the resulting rule would not satisfy even the weak incentives property.[43] This observation reveals an interesting subtle detail in the design of rules for environments such as ours. Specifically, if a central planner can *mandate which flights are to use which landing slots*, then various rules can satisfy our weak incentives requirement. On the other hand, if airlines are *free to rearrange* their own schedules (as mandated by current US procedures), then some rules (e.g. the FAA's current Compression algorithm) may lose this weak incentives property.[44]

Finally, it is straightforward to define analogous versions of Definition 14 that weaken *non-manipulability via weights* and *via slot-destruction*. It turns out that Theorems 2 and 3 can be strengthened by weakening the incentives conditions to these weaker versions. The proofs of this use precisely the same examples as the proofs of the current theorems.

[41] This is not quite a consequence of Theorem 4 since these rules are not defined to include self-optimization. The example in Subsection 5.4 can be used to prove the claim for both rules.

[42] Available in the online appendix.

[43] See online appendix.

[44] This observation is tied to the argument in Footnote 25, which uses a revelation principle argument to say that, when airlines have the right to determine their own subschedule, it is without loss of generality to restrict attention to self-optimized rules.

7 Summary

The FAA initially creates a landing schedule that assigns flights to landing slots at an airport. That initial landing schedule can become inefficient or even infeasible for various reasons, including flight cancellations, flight delays, or a change in the quantity of available landing slots due to changes in weather. Furthermore, some of the information needed to feasibly or efficiently create a new landing schedule is privately known by airlines. Thus airlines must be induced to report such information to a centralized authority (the FAA) who uses some reassignment rule to design a new, feasible landing schedule.

We analyze this naturally occurring market-design problem by separately considering the airlines' incentive to report three different kinds of relevant information: the delay of flights, the cancellation of flights, and the relative delay cost of each flight.

It is interesting to note that the FAA does not currently elicit information of the third kind (relative delay cost). Such information is necessary to determine how airlines can make Pareto-improving trades of slots. On the other hand, our first three results show that any Pareto-efficient reassignment rule would be subject to manipulation in *each* of three ways: misreporting flight delays, misreporting flight cancellations, or misreporting delay costs. Phrased differently, under any of these three relatively weak incentive conditions, one cannot hope to solicit relative cost information with the purpose of achieving full Pareto-efficiency.

For our next set of results, we strive only for a weaker efficiency condition of non-wastefulness. Such landing schedules can be constructed *without* knowing the airline's delay costs. Hence such information need not be reported to the mechanism (consistent with current practice of the FAA). Unfortunately such reassignment rules are also necessarily manipulable by misreporting flight delays. However, we show that a class of rules motivated by the Deferred Acceptance algorithm (DASO rules) *does* induce airlines to promptly report flight cancelations. This property is not held by any other reassignment rule considered so far for this application, including the FAA's current Compression algorithm. Finally, we show that DASO rules satisfy a weaker form of non-manipulability in flight delays defined by Schummer and Vohra (2012).

When consider this application, the most natural models that come to mind are those of trading indivisible objects, of constrained queuing, or of 1-sided matching à la Shapley and Scarf (1974). Yet, by interpreting it as a 2-sided matching problem in the spirit of Gale and Shapley (1962), we have derived a class of rules that best satisfy incentives properties among the rules that have so far been considered in this application. This interpretation is analogous to the way in which 1-sided School Choice models (Abdulkadiroğlu and Sönmez (2003)) are analyzed as if they are 2-sided College Admissions models. Our model mirrors the School

Choice model, in that the "College" side of the market (airlines) are the relevant agents, while the "students" (slots) are the objects being traded.

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A Online Appendix

This supplemental appendix is not for journal publication.

A.1 Airline preferences: substitutable but not responsive

Preferences in our paper are defined only over sets of a fixed cardinality. However, we show that we cannot imbed such preferences into "responsive preferences" over sets of any size, as defined in the college admissions literature.

Definition: A relation P defined over *all* subsets of slots is **responsive** when, for each $s, s' \in S$,

- for each $S' \subseteq S \setminus \{s\}$, we have $S' \cup \{s\} \succeq^w_A S'$ if and only if $s \; P \; \emptyset$; and
- for each $S'' \subseteq S \setminus \{s, s'\}$, we have $S'' \cup \{s\} \succeq^w_A S'' \cup s'$ if and only if $s \; P \; s'$.

The following example shows that some weight-based preferences over subsets of size $|F_A|$ are not consistent with any responsive relation over all subsets of S.

Example (Preferences of airlines are not responsive): Consider $S = \{1, 2, 3, 4, 5\}$, and let airline A have $F_A = \{f, f', f''\}$ with $e_f = 1$, $e_{f'} = 2$ and $e_{f''} = 3$, and with $w_f = 1.5$, $w_{f'} = 1$ and $w_{f''} = 8$. This induces the following preference ordering $\succ^w_A$ over subsets of size 3.

$\succ^w_A$
$1, 2, 3$
$1, 3, 4$
$\mathbf{1, 3, 5}$
$2, 3, 4$
$2, 3, 5$
$\mathbf{3, 4, 5}$
$1, 2, 4$
$1, 4, 5$
$\mathbf{2, 4, 5}$
$\mathbf{1, 2, 5}$

Let P be a preference relation over all subsets of S that coincides with $\succ^w_A$ on the above subsets. If P is responsive, then $\{1, 3, 5\} \succ^w_A \{3, 4, 5\}$ would imply $\{1\} \; P \; \{4\}$ (by letting $S'' = \{3, 5\}$ in the definition of responsiveness). Similarly, $\{2, 4, 5\} \succ^w_A \{1, 2, 5\}$ would imply $\{4\} \; P \; \{1\}$ (by letting $S'' = \{2, 5\}$). Since these conclusions are contradictory, P cannot be responsive.

In various proofs below we denote A's flights that can feasibly use s as

$$F_A^s \equiv \{f \in F_A : e_f \leq s\}.$$

For each airline A and each set of slots $T \subseteq S$, we say that T **is feasible for** A if there exist a (feasible) landing schedule Π such that $\bigcup_{f\in F_A} \Pi(f) \subseteq T$.[45]

The following requirement reflects the notion that if a slot is chosen from a large set $T' \subseteq S$, then it should still be chosen from within subsets of T'.

Definition 15. (e.g. see Roth (1984)) Preferences of an airline A, yielding choice function $C_A()$, satisfy **substitutability** when for each $T \subset T' \subseteq S$, with T feasible for A, we have $[T \cap C_A(T')] \subseteq C_A(T)$.

It is straightforward to show the following.

Proposition 8. *Preferences of airlines satisfy substitutability.*

Proof. Let $A \in \mathcal{A}$ and let $T \subset T' \subseteq S$ where T is feasible for A. Suppose that $s \in T \setminus s \notin C_A(T)$. We show $s \notin C_A(T')$ concluding the proof.

Since $s \notin C_A(T)$, the flights F_A^s all can be assigned to slots within T that are earlier than s. This implies that $F_A^s = F_A^{s-1}$ and $|\{\bar{s} \in T : \bar{s} < s\}| \geq |F_A^s| = |F_A^{s-1}|$.

Since $T \subset T'$ these inequalities imply $|\{\bar{s} \in T' : \bar{s} < s\}| \geq |F_A^s| = |F_A^{s-1}|$. That is, the flights F_A^s can be assigned to slots within T' that are earlier than s. Therefore $s \notin C_A(T')$. □

A.2 Slot-propose and Airline-propose deferred acceptance coincide

On our domain of problems, both the *slot-proposing* and *airline-proposing* versions of deferred acceptance yield the same outcome. In other words, the slot-optimal and airline-optimal stable matches coincide on our domain of landing slot problems. This equivalence is straightforward when one side of the market has a common ranking of agents on the other side of the market. While this common ranking does not hold in our model (due to the e_f's), there is "enough" commonality in their rankings for the result to hold. Indeed, any airline that utilize slot 1 agrees that it is in a sense a "best" slot (though not "the" best slot, as a highly weighted flight may ideally use some later slot). Therefore, stability will require slot 1 to go to the highest ranked airline that can use it. *Conditional on this*, a similar argument requires slot 2 to go its highest-ranked airline that can use it, etc.

[45]Note that this implies $|T| \geq |F_A|$.

Formalizing this, however, requires us to define an airline-proposing version of deferred acceptance that respects the initial landing schedule in the same way that DASO rules do in Step 0. Effectively, Step 0 is equivalent to modifying the priority orders $\gg$ so that each slot ranks its owner (under the initial landing schedule) highest. Indeed DASO rules could equivalently be defined this way. Here we define A-DASO rules using this convention. The algorithm is basically three parts: modifying the priorities, classic deferred-acceptance, and self-optimization as in DASO.

Definition 16. For a set of airlines $\mathcal{A}$ and profile of priorities $(\gg_s)$ on $\mathcal{A}$, the **A-DASO rule with respect to** $\gg$ associates with every instance $I \in \mathcal{I}(\mathcal{A})$ the landing schedule computed with the following "A-DASO algorithm."

> **Step 0:** (Owner has top priority.) For each slot s, let $\gg'_s$ be the priority order over airlines that satisfies (i) $s \in \Phi^0(A)$ implies that A is ranked first in $\gg'_s$, (ii) $s \notin \Phi^0(B) \cup \Phi^0(B)$ implies $[B \gg'_s C \Leftrightarrow B \gg_s C]$.
>
> **Step $k = 1$:** Each airline proposes to its favorite set of slots. Each slot s tentatively accepts the offer of its highest ranked proposer under $\gg'_s$, and rejects the other proposing airlines.
>
> **Step $k = 2, \ldots$:** If there were on rejections in the previous round, proceed to the Self-optimization step. Otherwise, *each* airline A proposes to its favorite set of slots from among those slots that have not already rejected A. (Note that by substitutability, A will re-propose to all of the slots that *accepted* its offer in the previous round.) Each slot s tentatively accepts the offer of its highest ranked proposer under $\gg'_s$, and rejects the other proposing airlines.
>
> **Self-optimization step:** For each airline A, assign A's flights to the slots who accepted its proposal in the previous step so that the resulting landing schedule is self-optimized. Break ties among equally-weighted flights by preserving their relative order in Π^0.[46]

Theorem 9. *For any priorities $\gg$ and any instance I, the outcomes of the DASO rule $\varphi^{\gg}(I)$ and the A-DASO rule associated with $\gg$ coincide.*

Proof. Fix priorities $\gg$, and suppose by contradiction that there is I such that $\Pi \equiv \varphi^{\gg}(I) \neq \varphi^{A-DASO,\gg}(I) \equiv \Pi'$. Let s be the earliest slot for which the rules differ: $s = \Pi(f)$ implies $\Pi(f) \neq \Pi'(f)$, and $\Pi(f) < s$ implies $Pi(f) = \Pi'(f)$.

[46]This tie-breaking is irrelevant as in DASO.

Let $\mathcal{A}_s$ be the set of airlines A that can feasibly assign some flight $f \in F_A$ to s *and* assign other flights in F_A to each slot $t < s$ that A receives under Π. It is obvious by feasibility that both DASO and A-DASO must assign to s a flight from an airline in $\mathcal{A}_s$. By Lemma 1, DASO gives s to the highest ranked airline in $\mathcal{A}_s$ under $\gg$.

Denote this highest-ranked airline as A and suppose A-DASO yields the set of slots $\Pi'(A)$ to A. By definition, it is clear that $s \in C_A(T \cup \{s\})$, i.e. A would choose to take s in exchange for some other slot assigned by A-DASO. But this means that under an airline-proposing version of DA, A would propose first to s before ultimately proposing to one of the other slots in $t > s$ that it ends up receiving. This means that s rejects A for one of the other flights in $\mathcal{A}_s$, contradicting the fact that A is highest-ranked in $\gg'$ among $\mathcal{A}_s$. □

A.3 Weak Incentives

Schummer and Vohra (2012) show that two simple rules—the FAA's Compression algorithm and the TC rule—satisfy weak non-manipulability via arrival times. Since their paper considers only simple rules and weak incentives, they can ignore the part of airline preferences represented here by weights w_f. Consequently they need not consider whether any landing schedule is self-optimized (since this is irrelevant when speaking of weak incentives).

Here we show that their incentives results are robust if we assume that the airlines (or the rule) first self-optimize the initial landing schedule. If a self-optimization step is prepended to the Compression algorithm (or the TC rule), then it is weakly non-manipulable via arrival times.

Proposition 10. *Consider the rule that first self-optimizes the initial landing schedule and then applies the Compression algorithm. This rule is weakly non-manipulable via earliest arrival times.*

The same conclusion holds for the rule that applies Schummer and Vohra's (2012) TC rule to a self-optimized initial schedule.

Proof. Let φ denote the rule that first self-optimizes the initial landing schedule and then applies the Compression algorithm. Fix an instance I, airline A, and flight $f \in F_A$. Suppose A misreports e_f to be $e'_f \neq e_f$. Let $I' = I_{e_f \to e'_f}$. Denote $\Pi = \varphi(I)$ and $\Pi' = \varphi(I')$.

Let Π_1 be the landing schedule that results from self-optimizing the initial landing schedule Π^0 using the parameters in I. Let Π'_1 be the landing schedule that results from self-optimizing Π^0 using the parameters in I'.

Suppose $\Pi_1 = \Pi'_1$, i.e. that A's misreport has no effect on the self-optimization of Π^0. Then the Compression algorithm is applied to two (optimized) instances that differ only in

e_f (and not in initial schedules). The result of Schummer and Vohra (2012) thus implies the result (since they take an arbitrary initial landing schedule as fixed and allow for arbitrary misreports).

Suppose $\Pi_1 \neq \Pi'_1$. Then the change of e_f to e'_f impacts the self-optimization exercise, so it must be that $\Pi_1(f) \neq \Pi'_1(f)$. We show that f ends up either with an infeasible slot or a later slot than it would without a misreport.

Case 1: $e'_f < e_f$.

Since Π'_1 is self-optimal for I' but not for I, it must be infeasible for I, i.e. $\Pi'_1(f) < e_f$. Since Compression never moves a flight to a later slot, $\Pi'(f) \leq \Pi'_1(f) < e_f$, i.e. f receives an infeasible slot. Therefore A does not benefit from this manipulation.

Case 2: $e_f < e'_f$.

Since Π_1 is self-optimal for I but not for I', it must be infeasible for I', i.e. $\Pi_1(f) < e'_f \leq \Pi'(f)$. Since Compression moves no flight to a later slot, $\Pi(f) \leq \Pi_1(f) < \Pi'(f)$, i.e. f gets a strictly later slot after the misreport.

In both cases, the misreport cannot improve the outcome of each of A's individual flights.

The proof is identical for TC. □

More generally, any rule that is weakly non-manipulable by arrival times remains so if the rule is augmented by first self-optimizing the initial landing schedule.

On the other hand, suppose such a rule *does not* self-optimize the initial schedule, but performs self-optimization after the rule operates. The following example shows that such rules can be manipulable in a strong way.

Example 1. This schedule is not self-optimized.

Slot	Flight	Airline	Earliest	Weight
1	$\emptyset$	C		
2	a_2	A	2	1
3	b_3	B	1	5
4	a_4	A	1	4

Since airline C has no flights, Compression assigns slot 1 to the earliest flight that can use it, namely b_3. Therefore A ends up with slots 2 and 3. Self-optimizing this allocation puts a_4 in slot 2, and a_2 in slot 3. If A misreports e_{a_2} to be 1, then Compression assigns a_2 and a_4 to slots 1 and 3. In actuality this is infeasible for a_2, but self-optimization at this point would put a_4 into slot 1 and a_2 into slot 3. This is a strong manipulation for A when self-optimization is performed only after execution of Compression.

The same manipulation in this example would benefit A if we apply a self-optimization step only after using the TC rule of Schummer and Vohra (2012). That rule prescribes the same outcome for instance I as Compression does. However, the manipulation by A would assign flights a_4 and a_2 to slots 1 and 2 respectively. That is, again A has a strong manipulation when self-optimization is performed only after execution of TC.